Dynamic Analysis of Series-Connected and Mechanically-Coupled Twin Synchronous Motor Drive

I. K. Onwuka[1*], G. C. Diyoke[1], E. S. Obe[2]

[1]Department of Electrical/Electronic Engineering, Michael Okpara University of Agriculture Umudike, Nigeria.
[2]Department of Electrical Engineering, University of Nigeria Nsukka, Enugu State, Nigeria.

ABSTRACT: Series-connection of the stator windings of electric motors could serve a number of purposes, including load balancing between two synchronous motors. This paper modeled and analyzed a drive system of two separate three-phase synchronous motors whose stator windings are series-connected by a unique stator winding scheme, and whose shafts are mechanically coupled to a common load shaft through a speed reduction gear driven through the pinions of the respective motors. The mathematical model is developed in detail, and the system is simulated using MATLAB/SIMULINK. It is observed that for the case of a balanced load on the respective shafts of the two motors, the dynamic behavior of the two motors are identical. It is further observed that with the particular stator winding arrangement giving rise to six-windings per motor unit, each motor is essentially a three-phase motor and may be operated direct on line (DOL).

I. INTRODUCTION

It is generally perceived that the induction motors are the work horse of the industries (Krause *et al*, 1995). This is largely due to the ruggedness, economy, and self-starting behavior of such motors. However, in large power applications, synchronous motors have often been preferred due to their capacity for higher power output per volume, power factor correction, and higher efficiency. Such motors may easily be found in grinding mills for cement and mineral ores (Seggewiss *et al*, 2014; Rodriguez *et al*, 2005; and Valentine *et al*, 1977).

Market demands for increased production saw the introduction of two-synchronous motor drives in grinding mills (Valentine *et al*, 1977). This was made necessary due to torque limitations on the teeth of the girth gear used to drive the mills. The drive arrangement will be as in Figure 1.
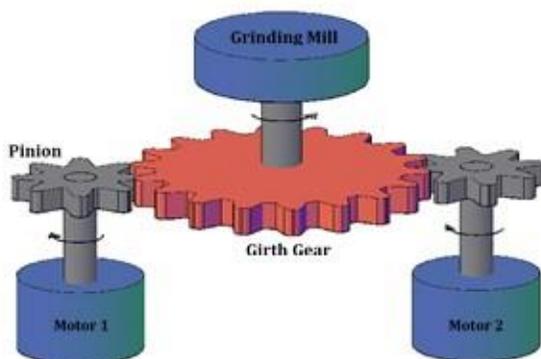
Fig. 1: Two Synchronous Motor Mill drive.

*Corresponding author: ifeanyikayonwuka@gmail.com

For such drive arrangements, it is important that the load on the girth gear is equally shared between the two motors. However, load imbalance has been observed, and is attributed to manufacturing errors and tolerance in the manufacture of the girth gear. The angular position of the motor shafts are observed to vary slowly, and the same with their angular speed, culminating in variation of torque produced by the respective motors. This has a deleterious effect on the gear life and often leads to motor over-heating. This problem has been resolved in a number of ways which are either expensive (Valentine *et al*, 1977; Scott and Valentine 1982), or leads to complex rotor circuit (Mular *et al*, 2002) or stator frame. A number of patents have also been obtained which propose solution to the load share problem between two synchronous motors coupled to a common shaft (Nelson, 1968; Kilgore *et al*, 1973; Ringland, 1980).

William Ringland (Ringland, 1980) proposed a system similar to the one under study. The startor windings of the two unit three-phase motors were split into two in the ratio 2:1. While the first part forms a set of three-phase windings in the first motor unit, the other part forms a set of three-phase windings in the second motor unit. In the end, each motor unit has two sets of balanced three-phase windings which are not identical. The present study considered a 1:1 stator winding split. The underlying hypothesis is that assuming identical machine properties for the two motors, the same electromechanical torque will be developed by the two motors if the current in motor *1* is the same current flowing in motor *2*.

Two-motor drive systems with series stator connections mounted on a common shaft have been studied (Anih and Obe, 2009; and Anih *et al*, 2014). The systems they studied are quite different from the one in the present study, as the motors were not

identical and they were mounted on a common shaft. Multi-phase Series-connected motors have also been studied (Jones *et al*, 2005; and Levi *et al*, 2007), with the aim of achieving independent vector control of the motors although they are fed from the same inverter. Onwuka and Obe (2019) analyzed the load sharing capability of a series-connected twin synchronous motor drive that were coupled to a common load shaft by gearing. However, in the present study, the aim is to model and analyze the behavior of two synchronous motors which are mechanically coupled to the same load through their respective pinions, and electrically connected to each other. Only the case of a balanced load is considered.

II. MATERIALS AND METHOD

A. System Description

Figure 2 shows the winding connection of the synchronous motors in the drive arrangement. Each phase of each motor is split into two winding sections with a turn ratio of 1:1, and the winding sections are designated s_1 and s_2 windings respectively. The s_1 windings of the first motor are connected in series with the s_2 windings of the second motor, and the s_1 windings of the second motor are connected in series with the s_2 windings of the first motor. It is pertinent to mention that there is zero angular displacement between the s_1 and s_2 windings of each motor, to maintain a three-phase system. The field windings of the two motors are connected in parallel.
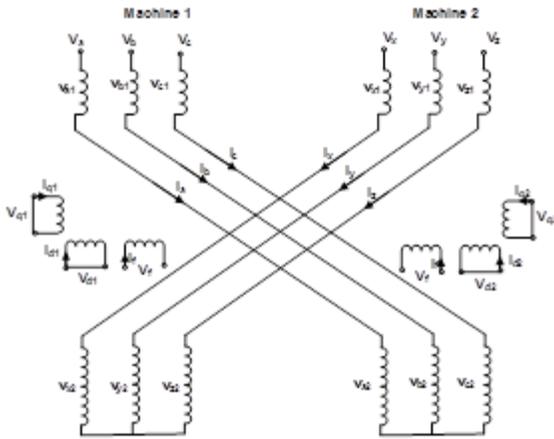
Fig. 2: Winding Connection of the two Synchronous Motors.

Moreover, this study assumes that the motors are electrically identical, and did not consider the non-linearity of the system. This system apparently renders each motor unit as a six-winding system. Six-winding systems have been studied by several authors (Levi *et al*, 2007; Palavicino and Valenzuela, 2015; Singh and Singh, 2012), and there study were relevant to the present study.

B. Modeling of the System Dynamics

As usual for any three-phase machine, applied voltages are given by:

$$V_n = V_m cos\left(\omega_e t - \frac{2\pi C}{3}\right) \tag{1}$$

Where $C = 0$, 1, or 2 for n = a, b, or c-phase, respectively.

As the $s1$ winding of one motor unit is in series with the $s2$ winding of the second motor unit, the voltages applied to the $s1$ terminals of one motor unit will supply the phase voltages in both the particular $s1$ winding of one motor and the corresponding $s2$ winding of the second motor. The assumption of electrical identity of the motors suggests that the applied voltage will divide between the $s1$ and $s2$ windings according to their turn ratio. As such, it will be observed from figure 2 that for any supply phase, *n*,

$$V_n = v_{n1} + v_{n2} = (r_{s1} + r_{s2})i_n + p(\lambda_{n1} + \lambda_{n2}) \tag{2}$$

Where the *r, i,* and λ terms refers to resistance, current, and flux linkage terms, respectively, and the subscripts *1* and *2* refers to the first and second winding sections of each phase respectively. In (2), *p* is the derivative symbol.

A six-winding machine has been implied from Figure 2. Previously, six-winding machines have been studied as six-phase machines (Singh and Singh, 2012). It is usual in such machines to refer the windings of one set of three-phase winding, say $s2$, to the windings of the other set of three-phase winding, say $s1$. This will modify (2) to become:

$$V_n' = r_s i_n + p\lambda_n' \tag{3}$$

where

$$V_n' = v_{n1} + v_{n2}' \tag{3a}$$

$$r_s = r_{s1} + \frac{N_{s2}}{N_{s1}}r_{s2}' \tag{3b}$$

$$\lambda_n' = \lambda_{n1} + \lambda_{n2}' \tag{3c}$$

where the *N* terms in (3b) refers to winding function terms of the specified winding, and the primed terms from (3) to (3c) refers to the variables referred from $s2$ to $s1$ windings. Moreover, as the non-linear equations in (2) contain time-varying inductance terms leading to computational complexity and plenty computer time for simulation, the use of Park's equations in the rotor reference frame is a common practice to simplify the solution to yield constant inductance terms. The twelve (12) resulting voltage equations describing the system is presented in compact matrix form in (4) and (5) as:

$$V'_{qdo1,2} = r_s i_{qdo1,2} + \omega_r \lambda'_{dqo1,2} + p\lambda'_{qdo1,2} \qquad (4)$$

$$V'_{qdr1,2} = r'_{qdr} i'_{qdr1,2} + p\lambda'_{qdr1,2} \qquad (5)$$

The flux linkages in (4) and (5) are expressed as follows:

$$\lambda'_{qdo1,2,r} =$$

$$\begin{bmatrix} \lambda'_{qdo1} \\ \lambda'_{qdo2} \\ \lambda'_{qdr1} \\ \lambda'_{qdr2} \end{bmatrix} \begin{bmatrix} \left(L_{11} + \frac{N_{s2}}{N_{s1}} L_{22}\right) & \left(\frac{N_{s2}}{N_{s1}} L_{12} + L_{12}\right) & L_{13} & L_{13} \\ \left(L_{12}{}^T + \frac{N_{s2}}{N_{s1}} L_{12}{}^T\right) & \left(\frac{N_{s2}}{N_{s1}} L_{22} + L_{11}\right) & L_{13} & L_{13} \\ L_{13}{}^T & \frac{N_{s2}}{N_{s1}} L_{13}{}^T & L_{33} & 0 \\ \frac{N_{s2}}{N_{s1}} L_{13}{}^T & L_{13}{}^T & 0 & L_{33} \end{bmatrix} \begin{bmatrix} i_{qdo1} \\ i_{qdo2} \\ i'_{qdr1} \\ i'_{qdr2} \end{bmatrix}$$

$$(6)$$

where the inductance terms L, in (6) are defined as follows:

$$L_{11} = \begin{bmatrix} L_{ls1} + L_{mq} & 0 & 0 \\ 0 & L_{ls1} + L_{md} & 0 \\ 0 & 0 & L_{ls1} \end{bmatrix} \qquad (7)$$

$$L_{22} = \begin{bmatrix} L_{ls2} + L_{mq} & 0 & 0 \\ 0 & L_{ls2} + L_{md} & 0 \\ 0 & 0 & L_{ls2} \end{bmatrix} \qquad (8)$$

$$L_{12} = \begin{bmatrix} L_{lm} & -L_{ldq} & 0 \\ L_{ldq} & L_{lm} & 0 \\ 0 & 0 & L_{laX} + L_{laY} \end{bmatrix} + \begin{bmatrix} L_{mq} & 0 & 0 \\ 0 & L_{md} & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad (9)$$

$$L_{13} = \begin{bmatrix} L_{mq} & 0 & 0 \\ 0 & L_{md} & L_{md} \\ 0 & 0 & 0 \end{bmatrix} \qquad (10)$$

$$L_{33} = \begin{bmatrix} L'_{lqr} + L_{mq} & 0 & 0 \\ 0 & L'_{ldr} + L_{md} & L_{md} \\ 0 & L_{md} & L'_{lfr} + L_{md} \end{bmatrix}$$

$$(11)$$

where the variables used in (7) – (11) are standard variables used for the q- and d-axes magnetizing inductances, and for leakage inductances. However, L_{laX} and L_{laY} defines the leakage inductances between the *a*- and *x*-phases, and between the *a*- and *y*-phases, respectively.

The currents in *qd0* variables may be obtained easily from (6). Since the two motors are not on the same shaft, the mechanical equations of the two motors are obtained separately. As such, the respective torque equation of the two motors will be:

$$T_{em1} = \frac{3}{2}\frac{P}{2}\left[\left(i_{q1} + \frac{N_{s2}}{N_{s1}} i_{q2}\right) L_{md}\left(i_{d1} + \frac{N_{s2}}{N_{s1}} i_{d2} + i'_{dr1} + i'_{fr1}\right) - \left(i_{d1} + \frac{N_{s2}}{N_{s1}} i_{d2}\right) L_{mq}\left(i_{q1} + \frac{N_{s2}}{N_{s1}} i_{q2} + i'_{qr1}\right)\right] \quad (12)$$

$$T_{em2} = \frac{3}{2}\frac{P}{2}\left[\left(\frac{N_{s2}}{N_{s1}} i_{q1} + i_{q2}\right) L_{md}\left(\frac{N_{s2}}{N_{s1}} i_{d1} + i_{d2} + i'_{dr2} + i'_{fr2}\right) - \left(\frac{N_{s2}}{N_{s1}} i_{d1} + i_{d2}\right) L_{mq}\left(\frac{N_{s2}}{N_{s1}} i_{q1} + i_{q2} + i'_{qr2}\right)\right] \quad (13)$$

The derivative of the angular speed of the two motors is given by:

$$p\omega_{r1} = \frac{P}{2J}(T_{em1} - T_{L1}) \qquad (14)$$

$$p\omega_{r2} = \frac{P}{2J}(T_{em2} - T_{L2}) \qquad (15)$$

The angular position of the rotors on their respective shafts will be:

$$\delta_1 = \int(\omega_{r1} - \omega_e)\, dt \qquad (16)$$

$$\delta_2 = \int(\omega_{r2} - \omega_e)\, dt \qquad (17)$$

To recover the currents in phase variables form, Eqn (18) is used:

$$i_{abcxyz} = \begin{bmatrix} T(\theta_r)^{-1} & 0 \\ 0 & T(\theta_r)^{-1} \end{bmatrix} \begin{bmatrix} i_{qd01} \\ i_{qd02} \end{bmatrix} \qquad (18)$$

Where:

$$T(\theta_r) = \frac{2}{3}\begin{bmatrix} \cos(\theta_r) & \cos\left(\theta_r - \frac{2\pi}{3}\right) & \cos\left(\theta_r + \frac{2\pi}{3}\right) \\ \sin(\theta_r) & \sin\left(\theta_r - \frac{2\pi}{3}\right) & \sin\left(\theta_r + \frac{2\pi}{3}\right) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \quad (18a)$$

and

$$\theta_r = \frac{(\theta_{r1} + \theta_{r2})}{2} \qquad (18b)$$

where θ_{r1} and θ_{r2} are the angular positions of the rotors of the two motors respectively.

The speed of the driven load will be obtained as:

$$\omega_m = \left(\frac{\omega_{r1} + \omega_{r2}}{2}\right) \times \frac{2}{P} \times \frac{1}{gr} \qquad (19)$$

where P is the number of poles of the machine, and gr is the gear ratio.

The parameters of the motors used is contained in Table 1, obtained by phase belt splitting of a three-phase synchronous motor.

Table 1: Parameters of the 18.65kW Synchronous Motors.

Parameter	Value
q-axis magnetizing reactance, X_{mq}	0.1789 Ω
d-axis magnetizing reactance, X_{md}	0.2919 Ω
Rotor q-axis leakage reactance, X_{lqr}	0.0720 Ω
Rotor d-axis leakage reactance, X_{ldr}	0.0692 Ω
Rotor field leakage reactance X_{lfr}	0.0759 Ω
Rotor q-axis resistance, r_{qr}	0.0109 Ω
Rotor d-axis resistance, r_{dr}	0.0115 Ω
Rotor field resistance, r_{fr}	0.0080 Ω
Stator resistance, r_s	$r_{s1} = 0.0222$ Ω $r'_{s2} = 0.0111$ Ω
Stator leakage reactance, X_{ls}	$X_{ls1} = 0.0193$ Ω $X'_{ls2} = 0.0141$ Ω
Leakage reactance between the two stator winding sets, X_{lm}	0.0058 Ω
Leakage reactance between the q- and d-axes of the 2 three-phase windings, X_{ldq}	0
Number of poles, P	6
Line voltage, V_L	208 V
Rated Current, I	57.8 A
Inertia of the rotating parts, J	0.08 kg-m^2
Field voltage	41 V

III. RESULTS AND DISCUSSION

Eqns (4) - (19) were implemented in MATLAB/SIMULINK environment. The elegance and simplicity of the embedded MATLAB function makes it the preferred tool of choice for the modeling of the global system. In this exercise, the motors were started at the same time on no-load, and after steady state occurred they were equally loaded with the rated load of 81 Nm at time 3

seconds. This rated load was determined by the current rating of the motor at 0.85 lagging power factor and field voltage of 41V. The important characteristics observed are as follows.

From Figure 3, it will be observed that the two motors attained steady state at the same time and have identical speed behavior.
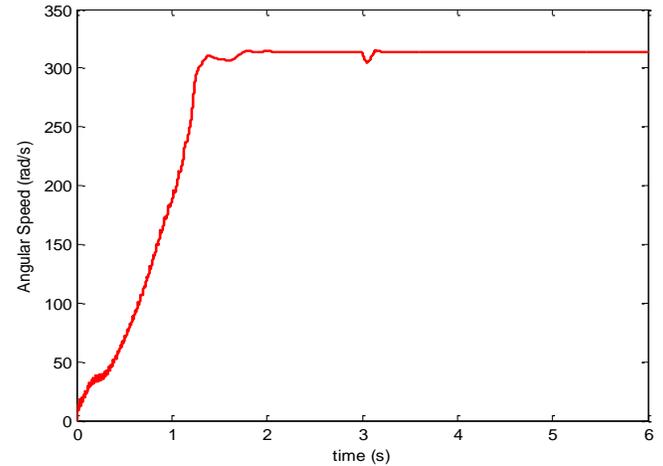
(a)
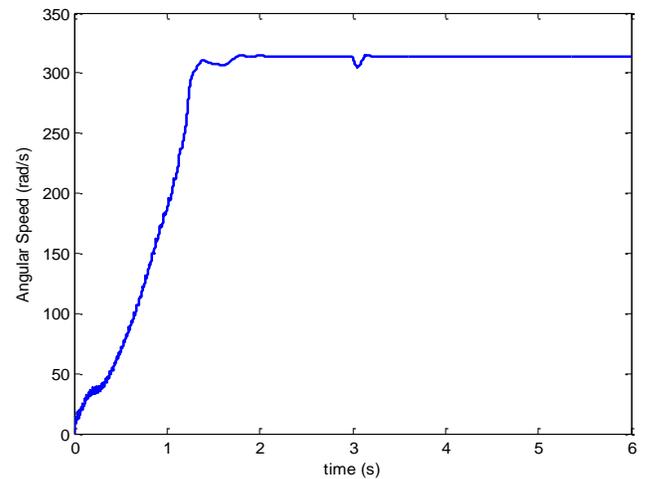
(b)

Figure 3: Angular Speed plotted against Time: (a) Motor 1, (b) Motor 2.
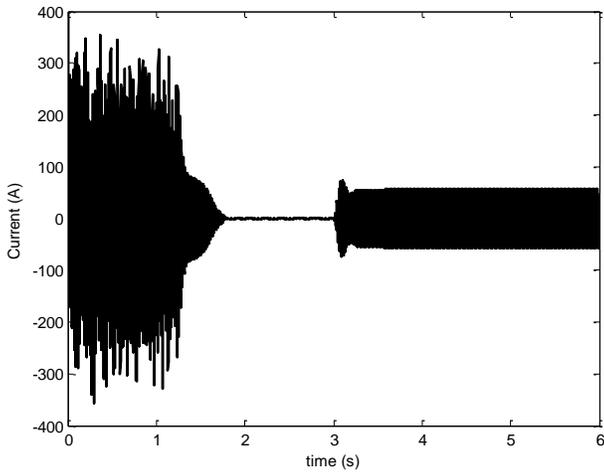
(a)
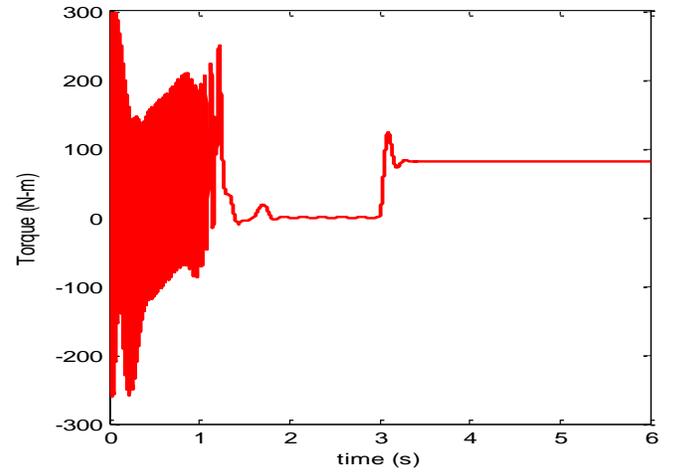
(a)
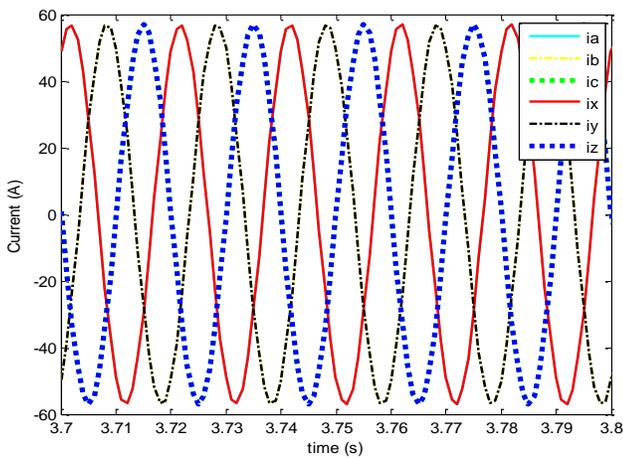
(b)
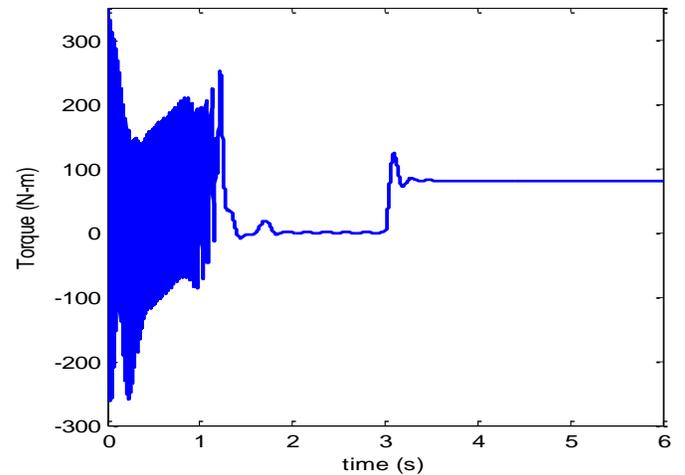
(b)

Figure 4: Phase Currents (common to the two motors), against time. (a) a-phase (b) The Six Winding Currents.

The stator currents are common to the two motors. The a-phase is shown in Figure 4a. Figure 4b captured the six winding currents in the same axis, where it is easy to see that indeed each motor is essentially a three-phase motor with six-windings. The x-phase coincided with the a-phase, the y-phase coincided with the b-phase, and the z-phase coincided with the c-phase.
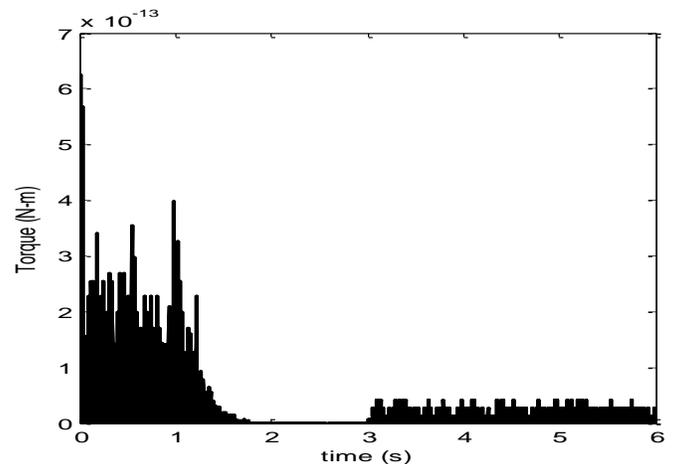
(c)

Figure 5: Torque Characteristic: (a) Motor 1 (b) Motor 2 (c) Torque difference between Motor 1 and Motor 2.

The torque characteristics of the motor is presented in figure 5. Expectedly, the developed torque of the two motors are identical in all respect, and after steady state, they reflected the load torque on their respective shafts.

Finally, the angular position of the rotors were observed and compared for any difference. No difference was observed, as captured in Figure 6.

IV. CONCLUSION

Two synchronous motors on separate shafts, whose stator windings are connected in series, and which are coupled by gearing to a common load shaft has been modelled and studied. When two identical synchronous motors have their stators connected together as in figure 2, the same current flows through the two motors and they have identical machine behavior, provided there are no predisposing conditions for load imbalance. While such stator winding arrangement involves six-windings, the motors are actually three-phase motors as the a-phase and x-phase variables coincide in every respect, as well as the b- and y-phases, and the c- and z-phases. As such, the motors maybe operated direct on line, or using any other starting technique available for three-phase motors. Such system may find application in high power applications as is obtained in mineral and cement grinding mills.

REFERENCES

Anih, L. U. and Obe E. S. (2009). Performance analysis of a composite dual-winding reluctance machine. Energy Conversion and Management, 50(12): 3056 – 3062.

Anih; L. U., E. S. Obe; and S. E. Abonyi. (2015). Modelling and Performance of a Hybrid Synchronous Reluctance Machine with Adjustable X_d/X_q Ratio. The Institution of Engineering and Technology (IET) Electric Power Applications. 9(2):171 – 182.

Jones, M.; S. N. Vukosavic; E. Levi; and A. Iqbal. (2005). A Six-Phase Series-Connected Two-Motor Drive with Decoupled Dynamic Control. IEEE Transactions on Industry Applications 41(4):1056 – 1066.

Kilgore, L. A., B. S. Strait Jr., E. C. Whitney. (1973). Twin Motor Drive System. United States Patent 3,735,217.

Krause, P. C; O. Wasynczuk; S. D. Sudhoff. (1995). Analysis of Electric Machinery. IEEE Press, New York, 1995.

Kunitoshi, K. (1985). Apparatus for Operating Plural Poly Phase AC Motors having a Common Load. United States Patent, 4,525,656.

Levi, E; M. Jones; S. N. Vukosavic; A. Iqbal; and H. A. Toliyat. (2007). Modeling, Control, and Experimental Investigation of a Five-Phase Series-Connected Two-Motor Drive with Single Inverter Supply. IEEE Transactions on Industrial Electronics, 54(3):1504 – 1516.

Nelson J. E. (1968). Control of Synchronous Motors on a Common Drive. United States Patent Office, 3,369,636.

Onwuka, I. K; and Obe E. S. (2019). Analysis of Load sharing capability of a series-connected twin synchronous motor drive coupled to a common load shaft by gearing. Nigerian Journal of Engineering and Technology, 38(2): 476 – 483.

Palavicino P. C., Valenzuela M. A. (2015). Modeling and Evaluation of Cycloconverter-Fed Two-Stator-Winding SAG Mill Drive – Part I: Modeling Options. IEEE Transactions on Industry Applications, 51(3):2574 – 2581.

Ringland W. L. (1980). Plural Eletric Motor Driving a Common Load and Having Interconnections for Power Control. United States Patent 4,208,620.

(a)

(b)

(c)
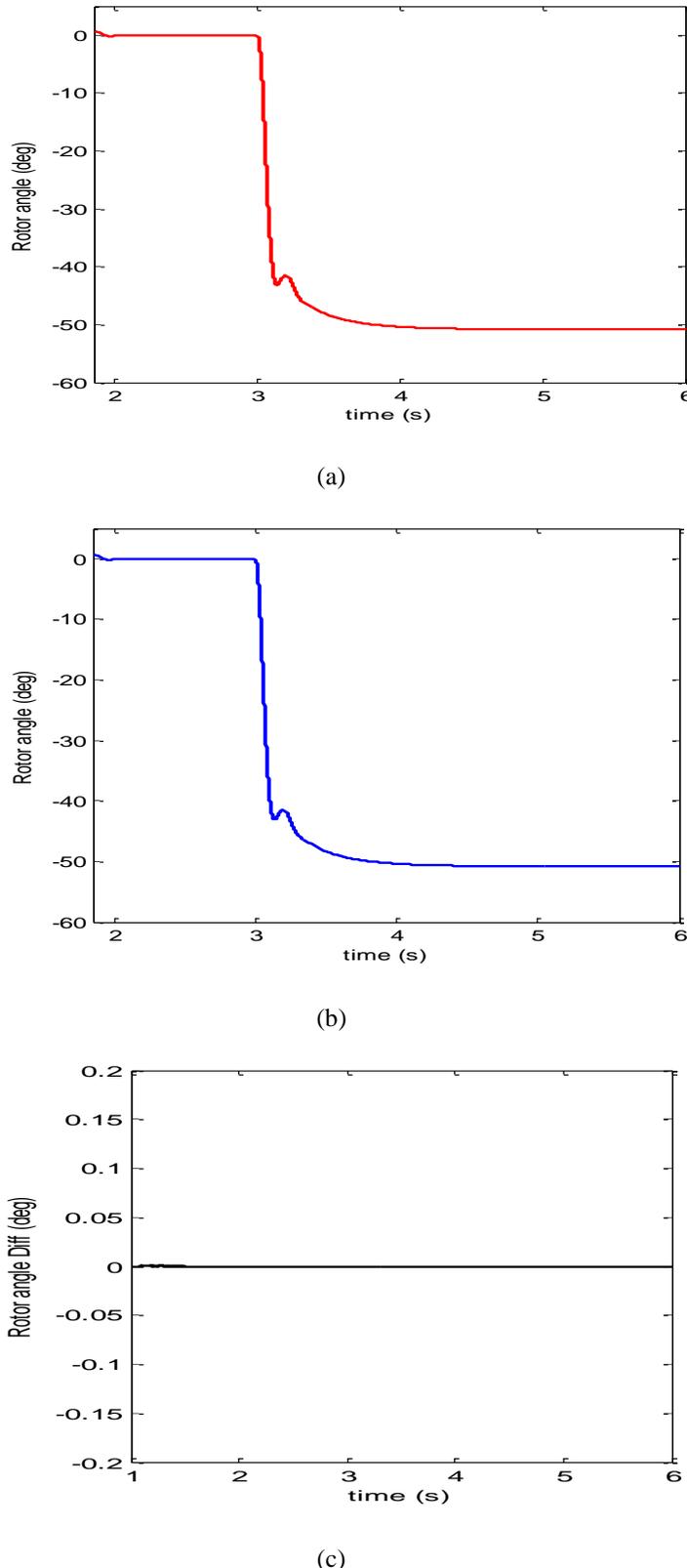
Figure 6: Rotor Angles: (a) Motor 1 (b) Motor 2 (c) Rotor Angle Difference between Motor 1 and Motor 2.

Rodriguez, J. R; J. Pontt; P. Newman; R. Musalem; H. Miranda; L. Moran; and G. Alzamora. (2005). Technical Evaluation and Practical Experience of High Power Grinding Mill Drives in Mining Applications. IEEE Transactions on Industry Applications, 14(3):866 – 874.

Scott, A. L. and Valentine, R. D. (1982). Large Grinding Mill Drives Update. IEEE Transactions on Industry Applications, IA-18(6):736 – 744.

Seggewiss, G; J. Dai; and M. Fanslow. (2014). Evaluation of Synchronous Motors on Grinding Mills. 2014 IEEE-IAS/PCA Cement Industry Technical Conference (CIC), National Harbor, MD, pp 1 – 12, USA.

Singh, G. K. and Singh, D. (2012). Experimental Study on Six-Phase Synchronous Alternator for Stand-Alone Renewable Energy Generation. IACSIT International Journal of Engineering and Technology, 4(4):344 – 347.

Valentine, R. D; J. G. Trasky; and D. R. Rippin. (1977). Load Sharing of Dual Motor Grinding Mill Drives. IEEE Transactions on Industry Applications, Vol. IA(2):161 – 168.