

Numerical Solution of Slug Flow using Differential Transform Method (DTM)



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ABSTRACT: Multiphase flow is a regular occurrence in different industries, including petroleum, chemical and nuclear industries. In the petroleum sector, multiphase transportation of fluids is usually confronted by flow assurance challenges as a result of flow physics, pipeline geometry or terrain through which the transport pipelines are laid. Slug flow is a flow assurance issue encountered in multiphase transport conduits. It is a regular pattern in the oil and gas production process and can significantly impair production if not properly predicted and controlled. In this work, using the differential transform method (DTM), a simple but yet robust approach is proposed for the numerical solution of slug flow models. The results showed slug flow properties were predicted with average absolute relative errors of $\pm 3-61\%$ accuracy compared to existing models with a range of $\pm 5-722\%$. This suggests that DTM offers a tremendous benefit of computational efficiency as well as accurate prediction of slug flow characteristics. Unlike previous models that rely on computationally intensive numerical simulations or empirical correlations with higher error margins, this study introduces the Differential Transform Method (DTM) as a more computationally efficient and accurate alternative solution technique for slug flow models. The novelty of this approach lies in its ability to provide closed-form approximations with reduced computational cost while maintaining a high level of accuracy. This study represents one of the first applications of DTM in multiphase flow modeling, demonstrating its potential as a viable alternative to conventional methods. Although the developed model performs creditably well in slug flow characteristics prediction, it may find application in slug flow control studies.

KEYWORDS: Multiphase Flow, Slug Flow, Differential Pressure, Flow Patterns, Flow Regimes, Differential Transform Method

SYMBOLS, ABBREVIATIONS AND UNITS

SYMBOLS

D - Pipe diameter (*ft*)

L_f - liquid flow ($\frac{ft^3}{m}$)

ρ_L - Liquid density ($\frac{lb}{ft^3}$)

ρ_G - Gas density ($\frac{lb}{ft^3}$)

S_{fr} - Slug frequency ($\frac{1}{s}$)

S_v - Mixture slip velocity ($\frac{m}{s}$)

V_F - Liquid film velocity relative to the bubble front ($\frac{m}{s}$)

V_G - Gas velocity relative to the bubble front ($\frac{m}{s}$)

θ - Angle of pipe inclination ($^\circ$)

g - Acceleration due to gravity ($\frac{m}{s^2}$)

τ_i - Interfacial shear stress (*Pa*)

τ_G - Wall shear stress of for the gas phase (*Pa*)

τ_F - Wall shear stress of for the liquid phase (*Pa*)

f_i - Fanning friction factor for the gas-liquid interface (dimensionless)

f_G - Fanning friction factor for gas-wall (dimensionless)

f_s - Fanning friction factor of the slug (dimensionless)

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f_L - Fanning friction factor for liquid-wall in the film region (dimensionless)
 Z - Axial section of the pipe (ft)
 h_f - Liquid film thickness (ft)
 H_F - Liquid holdup (dimensionless)
 h_s - Liquid film thickness at equilibrium (ft)
 H_S - Liquid holdup at equilibrium (dimensionless)
 A - Area (ft^2)
 S - Perimeter (ft)
 U_L - Average axial liquid velocity ($\frac{m}{s}$)
 U_T - Translational velocity ($\frac{m}{s}$)
 U_B - Bulk velocity ($\frac{m}{s}$)
 U_F - Velocity of Liquid Film ($\frac{m}{s}$)
 U_G - Gas velocity in film zone ($\frac{m}{s}$)
 U_M - Mixture velocity ($\frac{m}{s}$)
 C_o - Maximum per mean velocity of a fully formed velocity profile.
 ΔP_{FR} - Frictional pressure drop of the individual phases (Pa)
 ΔP_I - Pressure drop due to the interfacial effects (Pa)
 l_u - Slug unit length (ft)
 L_s - Slug length (ft)
 V_{sl} - Superficial liquid velocity ($\frac{m}{s}$)
 α - Dispersed phase length fraction (dimensionless)
 λ - Flowing liquid fraction (dimensionless)

ABBREVIATIONS

D - Diameter
 L_f - liquid flow
 S_F - Slug flow
 S_{fr} - Slug frequency

UNITS

Diameter - Feet (ft)
 Length - Feet (ft)
 Mass - Pounds (lb)
 Time - Seconds (S)
 Pressure - Pascal (Pa)

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I. INTRODUCTION

Multiphase flow is a complex and fundamental phenomenon in various engineering systems with attendant challenges. This phenomenon occurs in various manufacturing sectors including nuclear, oil and gas, chemical, transportation pipelines (Taitel and Barnea, 1990). In oil and gas industry, multiphase flow is majorly encountered in transportation of produced crude from reservoir to the topside. Multiphase slug flow could be primarily as a result of transporting more than a single phase in a pipeline or secondarily due to different and complex terrains. Different multiphase flow patterns occur in typical pipeline systems, but slug flow (S_F) is a major intermittent flow of concern with potential negative effect on the service life of pipeline and capacity. Irrespective of pipeline geometry, slug flow patterns may occur when suitable conditions exist. However, the slug flow characteristics may

vary contingent on the geometric configurations (Zheng *et al*, 1994; Ehinmowo, 2015).

In the 1970s, 80s and 90s, studies were focused on gaining fundamental insights into the phenomenon. Models were developed and experimental works provided data for the validation of developed models (Schmidt *et al*, 1985; Barnea, 1986). Later efforts were concentrated on investigating dynamic and transient behaviour of multiphase flow. Transient models were developed based on slug tracking and unit cell models (Schmidt *et al*, 1980; Barnea, 1986; Barnea and Taitel, 1993; Bendiksen *et al*, 1990; Zheng *et al*, 1994; Fabre and Line, 1992; Nydal and Banerjee, 1996; Taitel and Barnea, 1998).

Hill and Wood (1990) reported parameters that influenced slug formation and frequency among which are the height of the stratified liquid film in the section of slug development and the fluid slip velocity. A relationship describing slug frequency as a product of these two factors was developed. Tronconi

(1990) proposed a relationship between slug frequency and the characteristics of the vibrations that influence slug production based on Kordyban and Ranov (1970) and Mishima and Ishii (1980). The model developed was validated with a limited amount of data from small diameter horizontal conduits. However, the robustness may be limited and may not be applicable for flow in inclined pipes. Taitel and Dukler (1976) formulated an empirical correlation for estimating slug frequency.

Slug frequency (S_{fr}) is a major slug flow property. Although significant works exist on slug frequency, there is a high uncertainty associated with S_{fr} derived from mechanistic equations. Zabararas (2000) investigated the properties for slug flow in a conduit including slug frequency. Three hundred and ninety-nine data sets were obtained for a range of pipe diameters and inclinations. Archibong-Eso *et al* (2018) investigated slug frequency in different pipelines with distinct internal diameters using mineral oil, air and water as experimental fluids. A comparative evaluation of the correlations revealed that the developed correlation offered excellent results with an average absolute percentage error (AAPE) of 19.91%. Despite the excellent results, the study did not investigate other slug properties including liquid holdup, pressure gradient and slug length. Correlations between frequency and these properties offers a more comprehensive evaluation of slug behaviour across the pipeline for the design of effective slug control measures.

Other works including Al-Safran (2016) developed correlations for slug frequency estimation using data from dated works between late 80s and early 2000s. The proposed correlation can determine a pre-set probability range of S_{fr} predictions for a given flow operation. However, this work did not cover viscous fluids. Baba *et al* (2017) developed slug frequency for viscous fluid (mineral oil) comingled with air. Although the work addressed the issue of high viscosity (above 1 Pa. s) in horizontal pipes, behaviour in vertical pipes were not investigated. Whereas Zabararas (2000) reported that the error in available S_{fr} equations' predictions fell within a range of 75-115% for multiphase flow in different pipe configurations. The effect of pipeline configuration on slug frequency cannot be ignored.

Pressure gradient is also a significant slug flow property. Efficient determination of the pressure gradient is important for multiphase flow processes design and operations. The major critical operation in correlating pressure gradient involves evaluating the liquid holdup efficiently. In recent years, various correlations have been developed address different flow conditions. In the investigation by Luo *et al* (2020), various liquid holdup and superficial gas velocities were evaluated to develop a simple equation. The results revealed that the proposed correlation, can determine the pressure gradient of various pipe sizes excellently.

Other important characteristics including liquid holdup have been investigated and many methods have been developed in recent years for its measurements. The use of resistive probes ranked well for its simplicity and cost-effectiveness. Carraretto *et al* (2022) using traditional resistive

probe and an image-based method, explored liquid holdup measurements. The values obtained were in good agreement with those in available literature. However, resistive probes being intrusive devices may be prone to damage in high velocity pipeline operations.

Multiphase flow possesses a unique behaviour hence, efficient determination of the phase holdup and other flow properties can be challenging. Artificial Neural Network and Machine learning has been explored for the prediction of multiphase flow characteristics. Ehinmowo *et al* (2018) investigated riserbase pressure prediction using Artificial Neural network and the developed model shows accurate prediction compared to existing models. However Physically Guided Neural networks (PGNN) was not explored. Zhang *et al* (2024) explored PGNN for slug flow slug flow properties estimation including phase holdup but slug frequency and its influence in the characterization of slug slow was not investigated.

Kim and Kim (2022) studied multiphase flow and slug flow properties in a pipe. Tests were carried out using air and mineral oil. The evaluated pressure gradient data were also measured against investigations of previously developed equations for multiphase frictional pressure gradient. The new correlation efficiently determined the test pressure gradient data. However, other slug flow properties were not investigated. Liu *et al* (2022) investigated slug flow characteristics in a horizontal pipe. The slug frequency was reported to be influenced by the liquid and gas flow rates. But the flow condition had limited effect on the slug length. However, majority of the evaluated works were old.

A novel technique based on differential pressure signals was suggested by Guo *et al* (2023) to evaluate multiphase flow. Then, an equation was developed to relate the derived flow properties with the distinct phases. Test values showed that the developed technique and theoretical correlation are efficient and accurate. Both articles offered excellent results but did not consider enough recent articles for efficient comparison in order to verify the accuracy of their work. More recently, efforts are concentrated on modelling approach suitable for the enhancement of slug flow mitigation and control research. Al-safran *et al* (2004) developed a slug tracking system for identifying the characteristics for slug flow in a pipeline. Storkaas (2005) presented a comprehensive evaluation of the process properties that are required to control and mitigate slug flow. In the initial stage, a two fluid model was employed, and the evaluation revealed that riser slugging can be mitigated using topside valve.

Ogazi (2011) developed a comprehensive controllability analyses and strategies to obtain stability and maximum oil production from an unstable riser-pipeline. The method developed possesses the capability to stabilize the riser process at a valve which is greater than that obtained with the conventional model with about 4 % rise in production. Simões *et al* (2014) investigated multiphase flow system in a horizontal pipe utilizing the Two-Fluid correlation. The estimated pressure drop compared well with theoretical and existing experimental results on the other hand Zhao *et al* (2015) investigated highly viscous multiphase flow. Other

Researchers including Ehinmowo (2015); Ehinmowo *et al* (2016) proposed techniques for slug flow stabilisation. Ehinmowo (2015) developed a novel technique for slug flow characterisation and proposed both passive device and active feedback control strategies for slug flow mitigation. Results from the studies showed that slug flow intensity can be reduced up to 22% using a passive slug flow attenuation technique. Nnabuike (2019) proposed the use of Venturi device for slug flow mitigation and Doppler ultrasonic system for active slug flow mitigation system without negatively impacting oil production. Gao *et al* (2023) developed a gas hydrate transport correlation for a multiphase conduit in a deep-water gas field based on the two-fluid correlation combined with the particle size distribution correlation. Zhang *et al* (2025) developed a novel physics-informed neural network (PINN) for multiphase flow measurement. The outcomes revealed that PINN finds application across a wide range of conditions. Yang (2023) evaluated two distinct research works on carbon dioxide multiphase flow regimes in a near horizontal conduit. The work provided insights into the transition behaviour of multiphase flow. Cao *et al* (2022) investigated slug flow properties at the elbow of a pipeline system. The results showed that void fraction is location dependent. Peak values were observed at the inlet and the central locations experienced minimum values.

Although significant strides have been made in slug flow modelling, there is always the trade-off between accuracy, robustness and computational expense. Therefore, there is the need to employ numerical solutions that can achieve accurate solutions at less computational expense. Differential transform method (DTM) is a promising candidate that is investigated in this study. The Differential Transform Method (DTM) is a mathematical technique that leverages polynomial approximations to solve differential equations, facilitating the analysis of dynamic correlations in various processes. The theory of DTM was initially proposed by Zhou (1986), and the concept was applied to evaluate electric circuit systems. The technique provided a computational advantage over the Taylor series technique which requires extra computational work for higher orders. Abbasov and Bahadir (2005) employed generalized classical method (GCM), a technique based on DTM, for unsteady flow phase problems in integrated electrical and mechanical systems. The numerical solutions were compared with the analytical solutions of the models and the results showed good agreement. Odibat *et al* (2010) developed an advanced DTM technique with instantaneous and clear symbolic values of scientific application to differential equations. The technique showed less computational difficulties compared to other numerical techniques. Sobamowo *et al* (2020) evaluated the nonlinear oscillation of microbeam driven by electrostatic forces using the DTM technique. Yinusa *et al* (2023) investigated the deflection of double-walled Carbon Nanotubes (DWCNTs). models for the perpendicular movement of the CNT were derived, and solutions were obtained using Galerkin's decomposition technique and DTM. Same techniques were employed by Yinusa and Sobamowo (2023) for a system's stability under the influence of non-linear thermal, magnetic, and mechanical interactions and the oscillation of nanotubes

with a branched structure transferring magnetic nanofluid embedded in linear and nonlinear elastic models. Despite the versatility and robustness of this method, it has not been applied for multiphase flow in pipelines.

Despite significant strides in slug flow modelling, many existing methods either lack computational efficiency or with limited accuracy in varying pipeline conditions. This study aims to bridge this gap by employing the Differential Transform Method (DTM) to improve the prediction of slug flow characteristics, offering a balance between computational efficiency and accuracy. Specifically, this study will focus on validating DTM against experimental data to assess its performance in predicting slug frequency, pressure drop, and liquid holdup. DTM was used to solve slug flow models for the prediction of slug characteristics in pipelines. This article has been structured as follows: in the second section, the theoretical analysis is introduced. The materials and methods/methodology/experimental procedure section is represented in section three. The results and discussions of the DTM solution are shown in the fourth section. In the fifth section, the conclusion of the study is presented.

II. THEORETICAL ANALYSIS

Presented in this study is a correlation based on isothermal incompressible gas and liquid flow (L_f) (i.e., density is constant) and a steady flow is assumed by using a complex process operating at translational velocity (U_T). Hence the conservation correlations for the fluid phases are evaluated to derive the velocity-hold up profile of the slug tail. Following the schematic of a unit slug described in Figure 1, the mass conservation correlations for liquid and gas phase of the slug tail are given by equations 1 and 2.

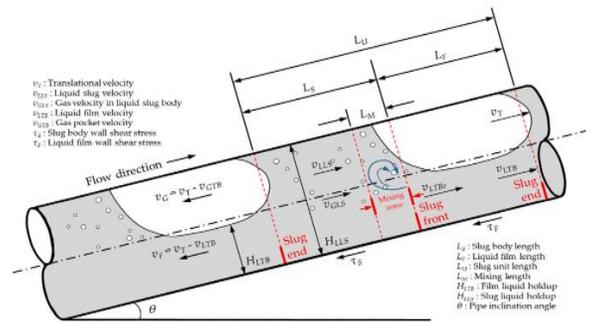


Figure 1: Representation of Slug Flow (Kim *et al*, 2020)

$$\rho_L \frac{\delta(A_F V_F)}{\delta z} = 0 \quad (1)$$

$$\rho_G \frac{\delta(A_G V_G)}{\delta z} = 0 \quad (2)$$

Where z , ρ_L , ρ_G , A_F , A_G , V_F and V_G are axial section of the pipe, liquid density, gas density, area occupied by liquid, area occupied by gas, relative velocities of the liquid and gas phases respectively.

For an observer moving at bubble nose propagating velocity, U_T , the continuity correlations of the slug unit could be described by equations (3) and (4):

$$H_S(U_T - U_L) = H_F V_F \quad (3)$$

$$(1 - H_S)(U_T - U_B) = (1 - H_F)V_G \quad (4)$$

Where U_L , U_B , H_S and H_F are average axial liquid velocity, bubble velocity, liquid holdup at equilibrium and liquid holdup respectively.

The momentum equation for fluid phases of slug were described by as equation (5 and 6) following Taitel and Barnea (1990), where $\frac{dp}{dt} = 0$:

$$\rho_L V_F \frac{\delta V_F}{\delta z} = -\frac{\delta P}{\delta z} + \frac{\tau_F S_F}{A_F} - \frac{\tau_i S_i}{A_F} + \rho_L g \sin \theta - \rho_L g \cos \theta \frac{\delta h_F}{\delta z} \quad (5)$$

$$\rho_G V_G \frac{\delta V_G}{\delta z} = -\frac{\delta P}{\delta z} + \frac{\tau_G S_G}{A_G} + \frac{\tau_i S_i}{A_G} + \rho_G g \sin \theta \quad (6)$$

$$V_F = U_T - U_F \quad (7)$$

$$V_G = U_T - U_G \quad (8)$$

Where P , θ , g and h_F are the pressure, angle of pipe inclination, acceleration due to gravity and liquid film height respectively.

In-line with the works Andreussi *et al* (1993) and Issa and Kempf (2003), the hydrostatic pressure variations or gradient ($-\rho_G g \cos \theta \frac{\delta h_F}{\delta z}$) is negligible due to much lower density of the gas, hence, in equation (6), the variation of h_F with z has been neglected. Assuming the pressure differential ($\frac{\delta P}{\delta z}$) present in the momentum correlation is negligible and substituting equations (7), (8), (A.5), (A.6) into equations (1), (2), (5) and (6) we obtain equations (9)-(12)

Recall that $\rho_L \frac{\delta(A_F V_F)}{\delta z} = 0$ and $\rho_G \frac{\delta(A_G V_G)}{\delta z} = 0$ from equations (1) and (2). The equations (A.5) and (A.6) are $A_F = AH_F$ and $A_G = A(1 - H_F)$, respectively. The correlations $V_F = U_T - U_F$ and $V_G = U_T - U_G$, are (7) and (8) respectively. A is the cross-sectional area of the pipe. When we substitute the values of equations (7) and (A.4) into equation (1) we obtain equations (9a-c) as:

$$\rho_L \frac{\delta(AH_F(U_T - U_F))}{\delta z} = 0 \quad (9a)$$

Taking into consideration that A is constant, equation (9a) becomes:

$$\rho_L A \frac{\delta(H_F(U_T - U_F))}{\delta z} = 0 \quad (9b)$$

The correlation can be re-written as:

$$\rho_L \frac{\delta(H_F(U_T - U_F))}{\delta z} = 0 \quad (9c)$$

Substituting equations (8) and (A.4) into equation (1) we obtain equations (10a-c) as:

$$\rho_G \frac{\delta(A(1-H_F)(U_T - U_G))}{\delta z} = 0 \quad (10a)$$

Taking into consideration that A is constant, equation (10a) becomes:

$$\rho_G A \frac{\delta((1-H_F)(U_T - U_G))}{\delta z} = 0 \quad (10b)$$

$$\rho_L H_F (U_T - U_F) \frac{\delta(U_T - U_F)}{\delta z} = T_w - T_n + \rho_L H_F g \sin \theta - \rho_L g \cos \theta H_F \frac{\delta h_F}{\delta z} \quad (11)$$

$$\rho_G (1 - H_F) (U_T - U_G) \frac{\delta(U_T - U_G)}{\delta z} = T_{gw} + T_n + \rho_G g \sin \theta (1 - H_F) \quad (12)$$

The wall temperature, gas-wall temperature and normal stress, respectively, are represented as:

$$T_w = \frac{\tau_F S_F}{A}, T_{gw} = \frac{\tau_G S_G}{A}, \text{ and } T_n = \frac{\tau_i S_i}{A} \quad (13)$$

The shear stresses between fluids in the film zone are highlighted in terms of absolute velocities given equation (14)-(16).

$$\tau_i = f_i \frac{\rho_G (U_G - U_F) |U_G - U_F|}{2} \quad (14)$$

$$\tau_G = f_G \frac{\rho_G U_G^2}{2} \quad (15)$$

$$\tau_F = f_L \frac{\rho_L U_F^2}{2} \quad (16)$$

where τ_i , τ_G , τ_F , f_i , f_G , and f_L are interfacial shear stress, wall shear stress of for the gas phase, wall shear stress of for the liquid phase, fanning friction factor for the gas-liquid interface, fanning friction factor for gas-wall, and fanning friction factor for liquid-wall in the film region, respectively. The velocity of liquid film and gas velocity in film zone are represented by U_F and U_G , respectively.

The friction factors of the fluids in contact with the wall could be evaluated using:

$$f = \frac{M}{Re^n} \quad (17)$$

Where r represents the various fluid phases and for smooth pipes, $n = 1$, $M = 16$ for laminar flow given that the Reynolds number (Re) is ≤ 2100 , & n is equal to 0.2, $M = 0.046$ for turbulent fluid movement if Reynolds number is larger than 2100. This is known as the McAdams formula for f . Several authors have utilized the Blasius formula which is $f = 0.0791/Re^{0.25}$. They have different ranges of application, where the McAdams's technique is wider, and more accurate when compared to Prandtl-Karman-Nikuradse's theory.

Geometrical parameters required to evaluate the Re for the fluid phases in the film region, is given in the Appendix. The Re for the fluid phases is defined as equations (18) and (19).

$$Re_F = \frac{4\rho_L A_F U_F}{S_F \mu_L} \quad (18)$$

$$Re_G = \frac{4\rho_G A_G U_G}{S_{(G+F)} \mu_G} \quad (19)$$

Re_F , Re_G , μ_G , and μ_L are Re for liquid phase, Re for gas phase, viscosity of gas phase and viscosity of liquid phase, respectively.

The Taylor bubble perimeter is a combination of the fluid mixture interface with that of the tube in contact with the Taylor bubble, hence the use of the term $S_{(G+F)}$, where G and F represent the gas and liquid phases, respectively.

The boundary conditions associated with mass and momentum equations is given as equation (20).

$$z = 0: U_F = U_M, U_G = U_M, H_F = H_S, h_F = h_s \quad (20)$$

Where h_s represents the liquid film thickness at equilibrium.

The translational velocity, U_T , is expressed as equation (21).

$$U_T = C_o U_M + U_d \quad (21)$$

Where slug mixture velocity is given as U_M , the maximum per mean velocity is C_o and drift velocity is expressed as U_d . Although, the general value for the constant C_o is not conclusive, an approximation was discovered to be 1.2. In the case of the drift velocity, U_d , the equation is represented as:

$$U_d = \frac{7}{20} \sqrt{gD} \sin \theta + \frac{27}{50} \sqrt{gD} \cos \theta \quad (22)$$

The drift-flux based approximation of dispersed bubble velocity in slug zone proposed is given as $U_B = 1.2U_M$.

Slug liquid holdup is referred to as a volumetric composition of liquid within the slug. Study of data from past literature has indicated that the slug H_F dependent on the fluid flowrates, conduit geometry, pressure and the fluid characteristics. Pereyra *et al* (2012) compared different mechanistic models and equations used for prediction of H_F against over 1400 experimental datapoints and they observed that predictive accuracy of Gregory *et al* (1978)'s equation is limited to near-horizontal pipes.

To solve the combined momentum equation and other model calculations, the value of holdup within the liquid slug is required and this will be determined utilizing the model suggested by Gregory *et al* (1978), due to its satisfactory prediction over a broad variety of application. The correlation is taken as equation (23)

$$H_s = \frac{1}{\left(1 + \left(\frac{U_M}{8.66}\right)^{1.39}\right)} \quad (23)$$

The mixture velocity is given as U_M .

The dependence of the Interfacial Friction Factor parameter on the behavior of the interface (smooth or wavy) between the fluids within the film zone makes it complex to predict, thus, the need to use of correlation to determine its value. In the case of model developed in this study, the Hagen-Poiseuille formula and the Taitel and Dukler (1976) correlation given as equation (24). Where laminar flow is represented in (24a), and turbulent flow is given by (24b).

$$f_i = \frac{16}{Re_i} \quad (24a)$$

$$f_i = \frac{0.046}{Re_i^{0.2}} \quad (24b)$$

The f_i formulas, in equation (24), were utilized for laminar and turbulent flow respectively. Where Re_i represents the interfacial Reynolds number given as equation (25).

$$Re_i = \frac{4A_G|U_G - U_F|\rho_G}{(S_G + S_i)\mu_G} \quad (25)$$

At the Reynolds numbers below 2000, the circulation is considered to be laminar, and when the Reynolds number is above 4000, the circulation is considered to be turbulent. In the transitional zone, between 2000 and 4000, the circulation is considered unsteady and could be laminar or turbulent. The Reynolds number (Re) combines the effects of several factors in a dimensionless factor.

Stable slug length observed in a slug flow process involving water-air indicates the independence of slug length with respect to flowrates the fluids and fairly dependent on the pipeline diameter (Taitel and Barnea, 1990). In this study, slug lengths of 16D and 32D (where D is the pipe diameter) is utilized for vertical and horizontal flow following the analysis of several researchers for low viscous fluid. For inclined flow, slug length correlation used by Zhang *et al* (2003a) model given below will be employed.

$$\frac{L_s}{D} = 32 \cos^2 \theta + 16 \sin^2 \theta \quad (26)$$

Where slug length is represented as L_s .

Differential Transform Method (DTM) was utilized to evaluate and develop the correlation. The DTM of an equation $z(t)$ is described as follows:

$$Z(k) = \frac{1}{k!} \left[\frac{d^k}{dt^k} z(t) \right]_{t=0} \quad (27)$$

Where $Z(k)$ is the transformed function $z(t)$. The inverse differential transformation of $Z(k)$ is highlighted as:

$$z(t) = \sum_{k=0}^{\infty} Z(k) * t^k \quad (28)$$

Substituting the value of $Z(k)$ defined in Eq.27 into Eq. 28, the DTM becomes:

$$z(t) = \sum_{k=0}^{\infty} \left[\frac{d^k}{dt^k} z(t) \right]_{t=0} * t^k \quad (29)$$

By using the equations 14 through 16, the basic operations of DTM functions can be summarized in the Table 1 below,

A few of the basic equations of DTM listed in Table 1 will be utilized in solution of the slug-tail velocity-holdup profile.

Table 1: Differential Transform of some functions

Function	Transformed Function
$z(t) = g(t) \pm h(t)$	$Z(k) = G[k] \pm H[k]$
$z(t) = \frac{dg(t)}{dt}$	$Z(k) = (k + 1)G[k + 1]$
$z(t) = \frac{d^m g(t)}{dt^m}$	$Z(k) = (k + 1)(k + 2) \dots (k + m)G[k + m]$
$z(t) = g(t) \cdot \frac{dg(t)}{dt}$	$Z(k) = \sum_{l=0}^k (l + 1)G[l + 1]G[k - l]$
$z(t) = h(t) \cdot g(t) \cdot \frac{dg(t)}{dt}$	$Z(k) = \sum_{p=0}^k \sum_{l=0}^p (l + 1)G[l + 1]G[p - l]G[k - p]$

Let the T-function of $U_F(x)$, $U_G(x)$, $H_F(x)$ & $h_F(x)$ be $F(k)$, $G(k)$, $H(k)$ and $L(k)$ respectively. Applying DTM to Eq. (9) and (10), we obtain

$$U_T(K + 1)H[K + 1] - \sum_{l=0}^k (l + 1)F[l + 1]H[k - l] - \sum_{l=0}^k (l + 1)H[l + 1]F[k - l] = 0 \quad (30)$$

$$\sum_{l=0}^k (l + 1)G[l + 1]H[k - l] + \sum_{l=0}^k (l + 1)H[l + 1]G[k - l] - U_T(K + 1)H[k + 1] - (k + 1)G[k + 1] = 0 \quad (31)$$

Similarly, Applying DTM to Eq. (11) and (12), we have,

$$\rho_L \sum_{p=0}^k \sum_{l=0}^p (l + 1)F[l + 1]F[p - l]H[k - p] - \rho_L U_T \sum_{l=0}^k (l + 1)F[l + 1]H[k - l] = T_w \delta[k] - T_n \delta[k] + \rho_L g \sin \theta H[k] - \rho_L g \cos(\theta) \sum_{l=0}^k (l + 1)L[l + 1]H[k - l] \quad (32)$$

$$\rho_G \left(U_T \sum_{l=0}^k (l + 1)G[l + 1]H[k - l] - U_T(k + 1)G[k + 1] + \sum_{l=0}^k (l + 1)G[l + 1]G[k - l] - \left(\sum_{p=0}^k \sum_{l=0}^p (l + 1)G[l + 1]G[p - l]H[k - p] \right) \right) = T_{gw} \delta[k] + T_n \delta[k] + \rho_G g \sin \theta (\delta[k] - H[k]) \quad (33)$$

$$\text{Where } \delta[k] = \begin{cases} 1, & k = 0 \\ 0, & k \neq 0 \end{cases}$$

The corresponding boundary conditions are transformed as

$$F[0] = U_L, G[0] = U_M, H[0] = H_s, L[0] = h_s \quad (34)$$

Utilizing the boundary conditions, Eq. (30) to (33) are solved simultaneously to obtain the iteration schemes for $F[k]$, $G[k]$, $H[k]$, $L[k]$ from $k = 0$ to $k = 5$. Substituting the iteration schemes into the main equation based on DTM we can obtain the slug velocity-hold up profile as follows.

$$U_F(z) = F[0] + F[1] \cdot z + F[2]z^2 + F[3]z^3 + F[4]z^4 + F[5]z^5 + F[6]z^6 + \dots \quad (34a)$$

$$H_F(z) = H[0] + H[1] \cdot z + H[2]z^2 + H[3]z^3 + H[4]z^4 + H[5]z^5 + H[6]z^6 + \dots \quad (34b)$$

III. MATERIALS AND METHODS/METHODOLOGY/EXPERIMENTAL PROCEDURE

A. Pressure Drop Calculation

The pressure gradient across the film zone and slug accounts for the pressure drop experienced within a slug body, in its entirety, with each pressure gradient having three contributing factors which include gravitational, frictional and acceleration pressure drop (P_d). The average P_{ds} ($\frac{\Delta P}{\Delta x}_u$) across the slug body is evaluated utilizing a momentum correlation over a slug defined as:

$$\left(\frac{\Delta P}{\Delta x}\right)_u = \left(\rho_s g \sin \theta + \frac{\tau_s \pi D}{A}\right) \cdot L_s + \rho_L H_s (U_T - U_L)(U_L - U_f) \quad (35)$$

For shear stress in the slug unit (τ_s), the equation is:

$$\tau_s = f_s \frac{\rho_s U_M^2}{2} \quad (36)$$

Where ρ_s is referred to as average density of the slug and f_s is the fanning friction factor of the slug. The ρ_s is calculated using:

$$\rho_s = \rho_L H_s + (1 - H_s) \rho_G \quad (37)$$

$$\mu_s = \mu_L H_s + (1 - H_s) \mu_G \quad (38)$$

$$Re_s = \frac{\rho_s U_M D}{\mu_s} \quad (39)$$

Applying this with values from the Gregory *et al* (1978) correlation helps us to derive the pressure gradient across the conduit, which is highlighted in the results.

A proposed pressure drop correlation by Jovanović *et al* (2011) was explained in two sections and the total P_d is represented in equation (40):

$$\Delta P_{slugflow} = \Delta P_{frictional} + \Delta P_{interfacial} = \Delta P_{Fr,c} + \Delta P_{Fr,d} + \Delta P_I \quad (40)$$

The frictional pressure drop, ΔP_{Fr} , was evaluated and represented in equations (41 and 42):

$$\Delta P_{Fr,d} = \frac{8\mu_d v \alpha l_u}{R^2} \quad (41)$$

$$\Delta P_{Fr,c} = \frac{8\mu_c v (1-\alpha) l_u}{R^2} \quad (42)$$

Where l_u is the slug unit length and the dispersed phase length fraction is represented as α .

The interface pressure ΔP_I is represented as:

$$\Delta P_I = \frac{2\gamma}{R} \cos \theta \quad (43)$$

Where γ represents the surface tension.

The correlation is applied in obtaining pressure drop values utilizing the Scott and Kouba (1990) as shown in section 4.

B. Slug Frequency Prediction

1) Tronconi model

Tronconi (1990) discovered that highlighted S_{fr} information in horizontal conduit and S_F could be determined with appropriate efficiency by estimating that the S_F is a part the S_{fr} of unsteady vibrations that are precursors of S_F . Utilizing the analytical evaluation of vibration instabilities Tronconi (1990) obtained the following correlation:

$$S_{fr} = 0.61 \frac{\rho_G v_G}{\rho_L (D - h_s)} \quad (44)$$

Equation 44 was used to derive the various S_{fr} data for this work. S_{fr} results have been obtained in a flow process evaluated. Several flow conditions were evaluated. This information was obtained with air, kerosene and water as experimental substances with various diameter conduits. Slug frequencies in the conduit were estimated by processing using available data, pipe characteristics, auxiliary equations and DTM. In the various diameter pipes, an array of slug dimensions and velocities for a particular collective of processing factors was discovered.

2) Gregory and Scott model.

Gregory and Scott (1969) evaluated S_{fr} s in a conduit for a CO₂/water process. Their equation is presented as:

$$S_{fr} = 0.0226 \left(\frac{V_{sl}}{gD}\right)^{1.2} \left[\frac{212.6}{U_M} + U_M\right]^{1.2} \quad (45)$$

The superficial liquid velocity is represented as V_{sl}

3) Babaa correlation.

Babaa *et al* (2017) conducted experiments utilizing a fluid mixture in a horizontal conduit. A high-speed Gamma Densitometer was utilized for data collection. For the variety of flow factors studied, a rise in oil viscosity was determined to effectively impact the SF. The resulting equation was given as:

$$S_{fr} = 1.8102 \frac{1}{N_f^{-0.4687}} \frac{V_{sl}}{D} \quad (46)$$

4) Archibong-Eso correlation.

Archibong-Eso *et al* (2018) investigated slug properties in two horizontal conduits. Test fluids utilized in the research were mineral oil and water with air as the gas phase. Results indicated that there exists a relationship between slug frequency and oil viscosity. A novel slug frequency equation was proposed

$$S_{fr} = 1.8102 \frac{1}{N_f^{0.4687}} \frac{V_{sl}}{D} \quad (47)$$

C. Experimental Data

The experimental study of Scott and Kouba (1990) and Gregory et al (1978) were utilized to verify the suggested model. The datasheets were chosen from Scott and Kouba (1990), and Gregory et al (1978) as a result of their availability and their extensive utilization in prior research works on multiphase flow. These datasets offer a well-documented reference for benchmarking novel correlations, enabling uniformity and consistency in comparative evaluation. Their utilization in multiple works reinforces their viability and allows for crucial comparisons between our proposed model and available experimental and theoretical approaches. Furthermore, these datasets cover a range of conditions particular to our work, making them appropriate for determining the accuracy of our estimations. In addition, their continued citation in modern research suggests their validity remains excellent. Table 2 shows the fluid properties, including densities and viscosities of air, kerosene and light oil, while Tables 3 represents the Experimental Data from Scott and Kouba (1990) and Gregory et al (1978).

Table 2: Fluid characteristics utilized in the model.

Fluid Properties	Air	Kerosene	Oil
Density	0.07516 $\frac{lb}{ft^3}$	49.9 $\frac{lb}{ft^3}$	53.56 $\frac{lb}{ft^3}$
Viscosity	$1.226 \times 10^{-5} \frac{f}{j}$	$1.6127 \times 10^{-3} \frac{j}{f}$	$4.535 \times 10^{-3} \frac{f}{j}$

The pipe diameter used in the Scott and Kouba (1990) is 0.25 ft in diameter and 0.085 ft for the Gregory et al (1978) experiment.

Table 3: Dataset employed in the model

Parameters	Mean	Minimum	Median	Maximum	Range	Standard Deviation	Variance
V_{SL} (meters/seconds)	0.634374	0.03	0.366	2.136	0.03-2.136	0.569267	0.324065
V_{SG} (meters/seconds)	2.083908	0.088	1.439	7.361	0.88-7.361	1.981697	3.927121
H_s	0.825299	0.39	0.89	1	0.39-1	0.156303	0.02443

The dataset highlighted in Table 3 are derived from Scott and Kouba (1990) and Gregory et al (1978), which evaluated Slug flow properties.

IV. RESULTS AND DISCUSSION

A. Slug Liquid Holdup Estimation

Several equations have been proposed for estimating H_F in horizontal conduits. Many of the correlations are derived

from experimental data and are limited in their applications. In addition, a lot of theoretical correlations with various magnitudes of difficulty have been put forward in existing works. Amidst them, the mechanistic equation formulated by Taitel and Dukler (1976) obtained excellent result. The bulk of the theoretical correlations developed afterwards are based on this. Many researchers analysed the Taitel and Dukler (1976) correlation and discovered that it was prone to underestimating holdup results. Apart from possible multiple roots, iterative process is required for its evaluation (Ogazi 2011).

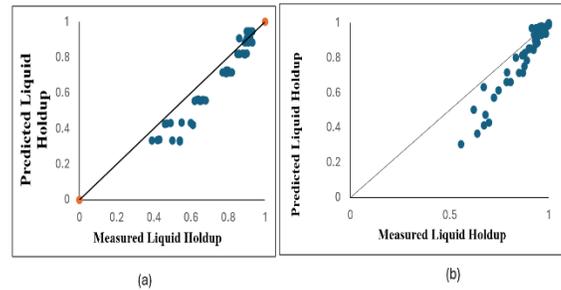


Figure 2: Slug liquid holdup: (a) Data from the Developed Model in comparison with Scott and Kouba (1990) (b) Data from the Developed Model in comparison with Gregory et al (1978)

A total of 112 data points, which were derived from the Scott and Kouba (1990) and Gregory et al (1978) data sets, were used to obtain comparative liquid holdup values for the predicted model. Knowledge of the of slug unit liquid holdup evaluations is significant for multiphase flow modeling, as deviations in estimation could result to substantial errors in pipeline design, pressure drop calculations, and flow assurance strategies.

In this study, the performance of the developed

correlation was evaluated by measuring its predicted slug liquid holdups against well-established experimental datasets, particularly those of Scott and Kouba (1990) and Gregory et al (1978).

The comparison in Figure 2a shows a strong agreement between the predicted slug unit liquid holdups from the developed correlation and the experimental data of Scott and Kouba (1990). The average relative error was found to be $\pm 11\%$, while a lower deviation of $\pm 2\%$ was observed when compared with Gregory et al (1978), as displayed in Figure 2b. These values show that the correlation displays a high degree

of accuracy in slug holdup prediction, especially when utilized to experimental datasets derived from near-horizontal flow conditions.

Despite this, the correlation's accuracy reduces with rising pipeline inclination and superficial velocities, which is in accordance with findings from previous research works (Taitel and Barnea, 1998; Taitel and Barnea, 1990). This decrease in accuracy can be attributed to the assumptions made during correlation development and the nature of the test data utilised to derive existing models. Stewart (2016) reported that errors in slug flow estimations can range up to $\pm 20\%$ for horizontal conduits and $\pm 50\%$ for inclined and looped procedures, proposing that deviations in the presented correlation remain within an appropriate threshold for practical utilizations. To further evaluate the correlation's efficiency, its evaluative accuracy was measured against various available models, as summarised in Table 4. The developed model obtained an average absolute relative error of $\pm 3\%$ to $\pm 11\%$, which is imperatively lower than that observed conventional correlations. By contrast, available correlations showed error margins ranging between $\pm 14\%$ and an extreme $\pm 722\%$, indicating significant deviations in particular situations.

A comprehensive evaluation of the outcomes in Table 4 displays different key insights: The developed correlation when measured against Scott and Kouba (1990) showed an average relative error of -10.52% and an average absolute relative error of 11.25% , with a standard deviation of 10.16% . Similarly, the developed correlation versus Gregory *et al* (1978) achieves a much lower average relative error of 2.04% , an absolute error of 3.39% , and a standard deviation of 4.90% , elucidating a superior accordance with this dataset. The measurement of Zhang *et al* (2003b) with Scott and Kouba (1990) outcomes showed an absolute relative error of 13.82% , with an increased standard deviation of 22.17% , proposing increased variability in its estimations. Abdul-Majeed (1996) versus Gregory *et al* (1978) presents an extreme error of 494.86% , with an even higher standard deviation of 656.15% , displaying imperative discrepancies in its prediction of slug liquid holdup. The Minami and Brill (1987) model when measured against Gregory *et al* (1978) performs even worse, achieving an extreme error of 722% with an 860% standard deviation, reinforcing the limitations of applying such empirical models outside their original development conditions.

The most accurate correlation after the developed model is Abdul-Majeed *et al* (2022) when measured against Gregory *et al* (1978), which generates an average relative error of 2.01% and an absolute relative error of 5.68% . Despite this, even this correlation shows increased error variability (standard deviation of 6.25%) when measured against the developed model, proposing that while it performs well under certain conditions, it is less reliable across a broader range of slug flow regimes. The outcomes demonstrate that the developed model provides a more stable and reliable estimation of slug unit liquid holdup, especially when measured against experimental datasets from available research works. Unlike previous models, which show excessive deviations under several flow conditions, the

presented correlation maintains an error range well within appropriate limits, making it a more viable tool for industrial utilizations.

Table 4: Comparison of the Results from the Model to Slug liquid Hold Ups of Various Works

Selected Model for Comparison	Average Relative Error (%)	Average Absolute Relative Error (%)	Standard Deviation of Average Relative Error (%)
Predicted model/ Scott and Kouba (1990)	10.5166	11.25283	10.16226
Predicted model/ Gregory <i>et al</i> (1978)	2.03	3.394	4.896232
Zhang <i>et al</i> (2003b)/ Scott and Kouba (1990)	-5.48135	13.8200979	22.16654
Abdul-Majeed (1996)/ Gregory <i>et al</i> (1978)	494.86	494.86	656.15
Minami and Brill (1987)/ Gregory <i>et al</i> (1978)	722	722	860
Abdul-Majeed <i>et al</i> (2022) / Gregory <i>et al</i> (1978)	2.00	5.68	6.25

B. Pressure Drop

A standard condition of $20\text{ }^{\circ}\text{C}$ was assumed in this research. The values achieved from the developed correlation were compared with experimental data from Scott and Kouba (1990) and Gregory *et al* (1978) data. Furthermore, our model was measured against several available correlations to further validate its predictive ability.

The comparative analysis of the predicted outcomes and theoretical results is shown in Figure 3, where it is observed that the overall prediction data align well with the expected theoretical values. One significant observation is the close clustering of data points across the graph, signifying a uniform and consistent distribution of errors across various flow conditions. The clustering shows that the correlation does not display large variations or outliers, reinforcing its stability and

reliability in determining pressure drop under diverse flow regimes.

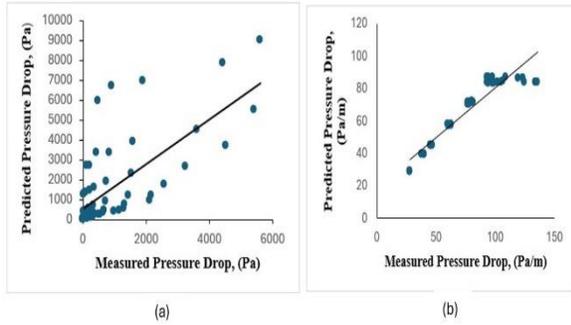


Figure 3: Pressure Drop: (a) Measured and Predicted Pressure Drop utilizing Values from Gregory et al (1978) (b) Measured and Predicted Pressure Drop utilizing Values from Scott and Kouba, (1990)

In multiphase flow modeling, randomly scattered points conventionally show high variance and poor model performance, whereas a close concentration of points around theoretical evaluations proposes a strong accordance. In our analysis, the observed clustering infers that the error margins stay within an acceptable limit across a wide variety of flow conditions, revealing the model’s robustness.

Table 5: Comparison of the Results from this Work to Pressure Drop of Various other Works

Selected Model for Comparison	Average Relative Error (%)	Average Absolute Relative Error (%)	Standard Deviation of Average Relative Error (%)
Predicted pressure drop/ Scott and Kouba (1990)	-10.5166	11.25283	10.16226
Predicted pressure drop/ Gregory et al (1978)	2.03763178	3.39444918	4.896232
Zhang et al (2003b)/ Scott and Kouba (1990)	-5.48135	13.8200979	22.16654
Morshed et al (2020)/ Gregory et al (1978)	-17.488	18.4055	13.270
Abdul-Majeed and Al-Mashat (2019)/Scott and Kouba (1990)	-23.726	23.7262	14.6378
Kora et al (2011)/ Gregory et al (1978)	4.79834619	4.861671714	7.405426573
Kora et al (2011)/Scott and Kouba (1990)	28.70863	28.708683	26.740893

Figure 3 also shows that deviations from theoretical values are slightly more pronounced in the high gas velocity regions, especially at increased void fractions, which is a traditional limitation of several pressure drop correlations. This is because, at increased gas flow rates, turbulence-induced fluctuations raise the uncertainty in both experimental and modeled results. However, the deviations remains relatively small, as shown in Table 5.

The presented model was compared to several experiments in Table 5. The average absolute relative error ranged between ± 5 to ± 29 whereas our developed model shows an enhanced performance with an error range between $\pm 3\%$ to $\pm 11\%$. This critical decrease in error proposes that our model offers better accuracy and reliability in comparison to previously established models. Several available models, exhibit increased errors at low flow rates as a result of underestimation of pressure drop in stratified flow conditions. The developed model, however, performs at an error margin of $\pm 3\%$ to $\pm 7\%$ in these conditions, revealing its capability to account for variations in liquid holdup and slug formation more efficiently.

Gregory et al (1978) and several other models tend to overestimate pressure drop in situations of increased gas velocities. This is majorly as a result of assumptions regarding interfacial shear and phase interaction forces. In contrast, our correlation provides a minimum and more stable error range of $\pm 5\%$ to $\pm 11\%$ in these conditions, suggesting a more effective representation of interfacial dynamics.

The peak error in our correlation ($\pm 11\%$) was seen in situations of very low liquid holdup, where experimental uncertainties are inherently higher as a result of difficulties in measuring phase interfaces efficiently. In comparison to alternative models, the proposed model consistently reveals a lower spread of errors, confirming its viability for a wide range of operational conditions.

C. Slug Frequency Estimation

Researchers revealed seven variables that they assumed had the most influence on slug frequency (S_{fr}). Hence, they showed the requirement for five dimensionless sets. Despite the large dataset which exist to the researchers, the wide variety of factors ensured that their evaluations became complex. No conclusive correlations were discovered amidst the five dimensionless sets, despite some tendencies becoming obvious. As a result of the lack of imperative improvements with the dimensionless evaluation path, the authors revealed a novel path for equating slug frequency core to the discoveries of the development of slugs in test rigs. The discoveries showed that both factors which had the major effect on the development of slugs were the height of the stratified L_f in the area of the development of slugs and the mixture slip velocity (S_v). Subsequently, the authors offered an equation showing S_{fr} as a function of these factors. Tronconi (1990) offered a relationship between S_{fr} and the characteristics of the vibrations that influence slug production. The author discovered that the SFr s during unsteady horizontal circulation were correlated to the periods of unsteady vibrations for the slug production in the entrance section of the conduit. The

author looked into the initiation of slugging by evaluating the vibration characteristics in relation to the concept of finite amplitude waves initially determined by several researchers. The final equation was accurately evaluated with a finite number of data from horizontal conduits. No relationships with inclined S_{fr} data were put forward. Taitel and Dukler (1976) produced a modified equation for investigating slug frequency. Despite their path being similar to many of the determined properties of slug generation, evaluation of both of the partial differential correlations defining the authors equation poses numerical concerns.

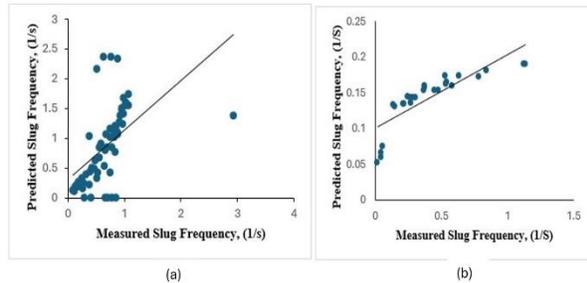


Figure 4: Slug liquid Frequency: (a) Measured and Predicted Slug Frequency utilizing Results from Gregory *et al* (1978) (b) Measured and Predicted Slug Frequency utilizing Results from Scott and Kouba (1990)

Table 6: Comparison of the Results from this Work to Slug Frequencies of Various other Works

Selected Model for Comparison	Average Relative Error (%)	Average Absolute Relative Error (%)	Standard Deviation of Average Relative Error (%)
Predicted slug frequency/ Scott and Kouba (1990)	-35.5997	61.4902	63.4100
Predicted slug frequency/ Gregory <i>et al</i> (1978)	-23.1521	24.1589	15.3603
Zhang <i>et al</i> (2003b)/ Scott and Kouba (1990)	-19.1000	31.80615	33.6164
Archibong-Eso <i>et al</i> (2018)/Gregory <i>et al</i> (1978)	-84.03	84.22	0.16
Sassi <i>et al</i> (2022)/ Gregory <i>et al</i> (1978)	-54.64	54.64	13.13

Accurate estimation of slug frequency plays a significant role in pipeline flow assurance, influencing parameters such as liquid holdup, pressure fluctuations, and pipeline integrity.

To validate the effectiveness of the developed correlation, comparative evaluation was conducted when measured against experimental datasets from Scott and Kouba (1990) and Gregory *et al* (1978). Furthermore, the correlation's performance was measured against other empirical and mechanistic models, such as those suggested by Zhang *et al* (2003b), Archibong-Eso *et al* (2018), and Sassi *et al* (2022). The outcomes of these comparisons, displayed in Table 6, offer key insights into the correlation's reliability, consistency, and overall performance relative to available approaches.

Figure 4 shows the overall accordance between the predicted slug frequencies and experimental datasets, where close clustering of outcomes is observed. This clustering proposes a consistent and minimal spread of errors, that is an indication of excellent predictive efficiency across the range of flow conditions taken into consideration. When measured against Scott and Kouba (1990), the correlation showed an average relative error of -35.60%, an absolute relative error of 61.49%, and a standard deviation of 63.41%. Against Gregory *et al* (1978), the correlation performed significantly better, with an average relative error of -23.15%, an absolute relative error of 24.16%, and a lower standard deviation of 15.36%.

The lower absolute relative error (24.16%) and standard deviation (15.36%) in comparison to Gregory *et al* (1978) proposes that the correlation is more viable when applied to horizontal pipeline conditions, where slug flow properties are more predictable. The increased absolute error (61.49%) and standard deviation (63.41%) for Scott and Kouba (1990) indicate that the correlation encounters greater deviations in inclined pipeline conditions, likely as a result of increased gravitational and inertial forces influencing slug development.

In addition to validating the correlation against test datasets, a comparative evaluation was carried out with other slug frequency correlations. The error outcomes from these comparisons are summarized in Table 6, where different notable trends emerge. The correlation by Zhang *et al* (2003b), when measured against Scott and Kouba (1990), showed an absolute relative error of 31.81% and a standard deviation of 33.62%.

While this error range is decent, it is still higher than that observed in the developed correlation when measured against Gregory *et al* (1978). This suggests that while Zhang's correlation has some predictive accuracy, it still shows considerable variability, making it less appropriate for precise slug frequency evaluations. This correlation performed poorly, with an absolute relative error of 84.22%. Interestingly, its standard deviation was only 0.16%, indicating that while the correlation was consistently inaccurate, its predictions deviated systematically in a particular direction rather than fluctuating unpredictably.

The correlation by Sassi *et al* (2022) generated an absolute relative error of 54.65% with a standard deviation of 13.14%. This shows that while Sassi's correlation was less accurate overall, it had lower variability than Zhang *et al* (2003b). The presented correlation's average absolute relative error of ± 24

to ± 61 makes it a very viable correlation for accurately determining pressure drop in comparison to those highlighted in Table 6.

V. CONCLUSION

This study provides a comprehensive analysis of slug flow properties, focusing on slug unit liquid holdup, slug frequency, and pressure differential estimations utilizing a refined two-fluid model solved with the Differential Transform Method (DTM). The accuracy and viability of the solutions were evaluated by measuring its predictions against experimental datasets from Scott and Kouba (1990), Gregory *et al* (1978), and other established research works. The correlation revealed strong agreement with experimental datasets, achieving an average relative error of $\pm 11\%$ (Scott and Kouba, 1990) and $\pm 2\%$ (Gregory *et al*, 1978). Prediction accuracy reduced with rising pipeline inclination and superficial velocities, attributed to assumptions in correlation development and dataset properties. In comparison to available correlations, the formulated model showed superior accuracy, achieving an absolute relative error of $\pm 3\%$ to $\pm 11\%$, whereas alternative correlations displayed errors ranging from $\pm 14\%$ to $\pm 722\%$.

The correlation's average absolute relative error ranged from $\pm 24\%$ to $\pm 61\%$, making it a crucial tool for slug frequency estimation, particularly under horizontal flow conditions. Comparative evaluations against existing correlations confirmed that the formulated correlation achieved more uniformity and reliable estimations. However, higher deviations were revealed for inclined and looped pipeline conditions, proposing a need for further refinements to address gravitational and inertial effects.

The research article also analysed the pressure differentials across the slug flow, highlighting the correlation's efficiency in estimating pressure drop behaviour. The correlation's estimations aligned closely with theoretical expectations and experimental datasets, with an absolute relative error of $\pm 3\%$ to $\pm 11\%$ —a significant improvement over alternative correlations. The efficiency of pressure differential predictions reduced with increasing superficial gas velocity, an observation uniform with literature reports on slug flow dynamics.

The findings from this work demonstrated the practical importance of the developed correlation in multiphase flow processes, especially in pipeline flow assurance and pressure drop estimation.

Despite its excellent predictive ability, the solutions showed increased errors in inclined and looped flow conditions, proposing that additional correction factors may be required for improved accuracy under such scenarios.

To additionally refine and advance the applicability of the correlation, future studies may introduce machine learning-based corrections for more adaptability across various flow conditions, refine correction factors for non-horizontal conduits, address the influence of gravitational forces on slug dynamics, expand validation datasets, incorporate a wider range of conduit diameters, pressure conditions, and fluid characteristics, and investigate turbulence and interfacial shear effects, which play an important role in pressure differential fluctuations.

AUTHOR CONTRIBUTIONS

A. B. Ehinmowo: Conceptualization, Methodology, Software, Validation, Writing – original draft, Supervision, Writing – review & editing. **D. A. Vershima:** Writing – original draft, Writing – review & editing. **A. Akinniyi:** Writing – review & editing. **O. Adewumi:** Supervision, Writing – review & editing. **I. Adewale:** Supervision, Writing – review & editing. **O. D. Orodu:** Supervision, Writing – review & editing. **A. M. Aliyu:** Supervision, Writing – review & editing.

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APPENDICES

Appendix A. Geometrical parameters

Where A_F , A_G , S_G , S_F and S_i , H_F are geometric variables, which are functions of liquid film thickness (h_f) defined as:

$$\lambda = 2 \cos^{-1} \left(1 - \frac{2h_f}{D} \right) \quad (\text{A.1})$$

Where the flowing liquid fraction is represented as λ

$$S_F = \frac{D\lambda}{2} \quad (\text{A.2})$$

$$S_i = D \sin \left(\frac{\lambda}{2} \right) \quad (\text{A.3})$$

$$S_G = D \left(\pi - \frac{\lambda}{2} \right) \quad (\text{A.4})$$

$$A_F = AH_F \quad (\text{A.5})$$

$$A_G = A(1 - H_F) \quad (\text{A.6})$$

$$H_F = \frac{\lambda - \sin(\lambda)}{2\pi} \quad (\text{A.7})$$

Appendix B. Statistical parameters

Statistical parameters used to evaluate the performance of the model with measured data are based on average relative error and average absolute relative error:

$$e_{ri} = \frac{E_{i,cal} - E_{i,measured}}{E_{i,measured}} \times 100 \quad (\text{B.1})$$

$$e = \frac{1}{N} \sum_{i=1}^N e_{ri} \quad (\text{B.2})$$

$$\varepsilon = \frac{1}{N} \sum_{i=1}^N |e_{ri}| \quad (\text{B.3})$$

Standard deviation related to average relative error is:

$$\varepsilon = \sqrt{\frac{\sum_{i=1}^N (e_{ri} - e)^2}{N-1}} \quad (\text{B.4})$$