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GENETIC ALGORITHM-BASED PRECISION TUNING OF DIGITAL P-I-D CONTROLLER FOR SECOND-ORDER SYSTEMS

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ABSTRACT

Tuning of the PID controller is often done by trial and error which is tedious, time consuming and relatively inefficient. A more reliable and efficient approach of tuning the PID controller using genetic algorithm is presented. The scope of this work is limited to obtaining control parameters of a digital PID controller for a second-order system, given the desired time-response parameters.
Keywords: Digital PID Controller Tuning, Second-order system, Genetic Algorithm.

1.0 INTRODUCTION

A controller is usually incorporated in a control system to improve the overall performance of the system. The controller of interest is the digital Proportional-Integral-Derivative (PID) controller and the control systems of interests are the second order systems. A PID is the most common form of feedback in use today and is widely used in industrial control systems to improve the overall efficiency of the system. It attempts to correct the error between a measured process variable and set point by calculating and outputting a corrective action that can adjust the process accordingly. This paper is based on optimal tuning of a digital PID controller for second order systems using Genetic Algorithm (GA). It takes a look at an existing method of tuning a PID controller, which is usually trial-and-error , and the development of a new method based on GA.

1.1 Standard Second-order System

A second order system is one whose characteristic equation is of second order with the closed-loop transfer function given by equation (1):

$$\frac{C(s)}{R(s)} = \frac{w_n^2}{s^2 + 2\xi w_n s + w_n^2} \qquad (1)$$

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The time response is characterized by the roots of the denominator polynomial, which are the poles of the transfer function (Nagrath and Gopal, 2005). The denominator polynomial is called the characteristic polynomial and the characteristic equation is given by equation (2)

$$s^2 + 2\xi w_n s + w_n^2 = 0 \qquad (2)$$

The roots of the characteristic equation (i.e. the poles) are given by equation (3) with the solution in equation (4):

$$s^2 + 2\xi w_n s + w_n^2 = (s - s_1)(s - s_2) \qquad (3)$$

$$s_1, s_2 = -\xi w_n \pm j w_n \sqrt{1-\xi^2} = -\xi w_n \pm j w_d \qquad (4)$$

where $w_d = w_n \sqrt{1-\xi^2}$, is called the damped natural frequency.
When $0<\xi<1$, the poles from a complex conjugate pair and th e system is said to be underdamped. The system breaks into continuous oscillation when $\xi = 1$ and becomes overdamped for $\xi > 1$.

1.2 Time Response Specifications of Second Order Systems

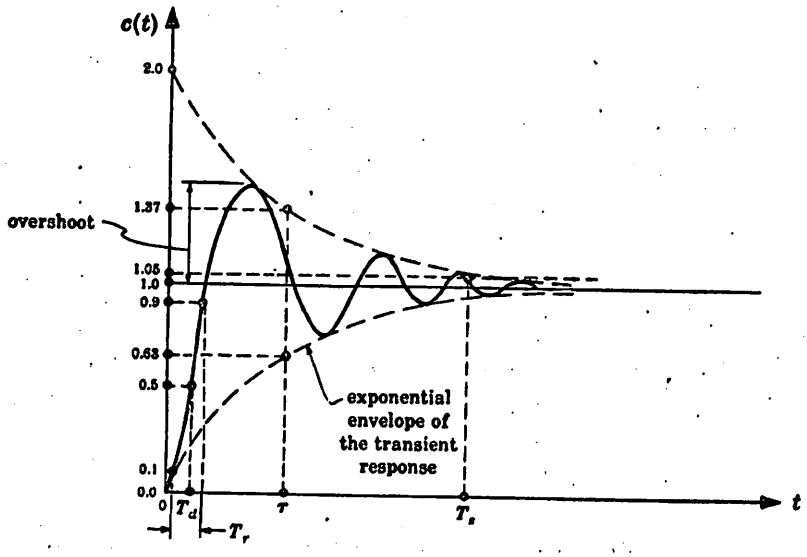

Figure 1: Continuous time response of a second order system to a unit step input (Nagrath and Gopal, 2005)

The response of a second order system in time domain (e.g. to a step input) is generally damped oscillatory in nature as shown in Figure 1 (Nagrath and Gopal, 2005).

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The degree of oscillation of the system usually depends on the damping factor (). Oscillation is continuous at . The system becomes less oscillatory as is increased and just non-oscillatory for. From Fig.1, Peak overshoot, rise time, peak time, settling time and steady state error are as shown. These parameters are known as the time response specification of the system

2.0 CONTINUOUS-TIME CONTROLLERS/COMPENSATORS

The design of a control system is an attempt to meet a set of specification which defines the overall performance of the system in terms of some measurable quantities known as the time response specification (Nagrath and Gopal, 2005). Given a control system with a set of specification in terms of peak overshoot, rise time, peak time, settling time and steady state error, controllers are designed so that the system meets the given specification. The input to the controller is the error signal resulting from the difference between the systems input and output as shown in the Figure 2. The controller gives an output, based on the input error signal, which becomes the corrective input to the plant.

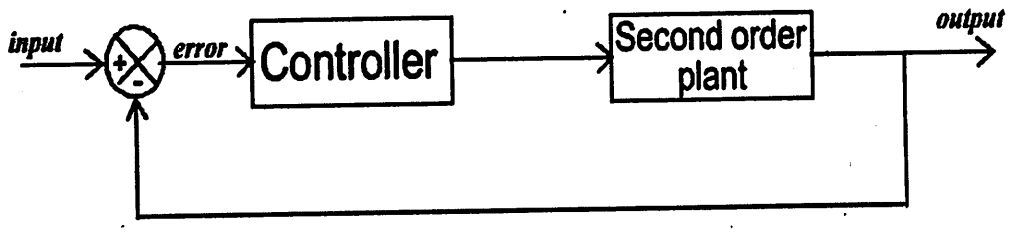

Figure 2: A Control system

Proportional-Integral-Derivative (PID) Controller:

The output of this controller consists of three terms, one proportional to the error signal itself, the second proportional to the Integral of the error signal and the third directly proportional to the derivative of the error signal. That is,

$$u(t) = K_p e(t) + K_i \int_{-\infty}^{t} e(t)dt + K_d \frac{de(t)}{dt} \qquad (5)$$

Equation (6) is the transfer function of a PID controller where the proportional constant is, is integral time, is derivative time, K_i is integral constant and K_d is derivative constant.

$$G_c(s) = K_p\left[1 + \frac{1}{\tau_i s} + s\tau_d\right] = K_p + \frac{K_i}{s} + K_d s \qquad (6)$$

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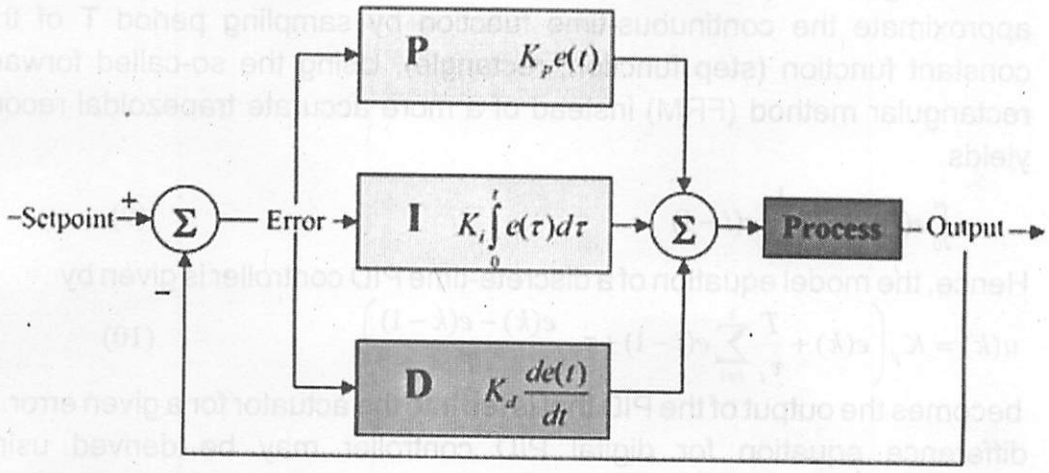

Figure 3: PID controller

The time response specifications (peak overshoot, rise time, etc) are not independent of each other and, so, the P, PI or PD controller, sometimes, cannot successfully modify the behaviour of the control system as desired. For example, a PI may successfully eliminate steady state error but also make the system unstable with a high peak overshoot, to increase the damping factor and make the system stable; the derivative error controller is added to make it a PID controller. Optimal control of a system or plant may be obtained by using a properly tuned PID controller.

2.1 Digital PID Controller

Discretization of a continuous-time PID controller is used to obtain a digital PID controller. A continuous-time PID controller model in time-domain may be written in the form

$$u(t) = K_p\left(e(t) + \frac{1}{\tau_i}\int_{-\infty}^{t} e(t)dt + \tau_d \frac{de(t)}{dt}\right) \quad (7)$$

To obtain the equation of the digital version of a continuous-time PID controller we must discretize the integral and derivative components of equation (16) (Bobal et al., 2005). When the sampling period T is small and noise from the process output signal is effectively filtered out, the simplest algorithm is obtained by replacing the derivative with a difference of the first-order (two-point, backward difference).

$$\frac{de(t)}{dt} \approx \frac{e(k) - e(k-1)}{T} \quad (8)$$

Where is the value of the error at the kth moment of sampling at time t=kT.

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The integral component is approximated by simply summing so that we approximate the continuous-time function by sampling period T of the constant function (step function, rectangle). Using the so-called forward rectangular method (FRM) instead of a more accurate trapezoidal record yields

$$\int_0^t e(t)dt \approx T\sum_{i=1}^{k} e(i-1) \tag{9}$$

Hence, the model equation of a discrete-time PID controller is given by

$$u(k) = K_p\left(e(k) + \frac{T}{\tau_I}\sum_{i=1}^{k} e(i-1) + \tau_d \frac{e(k)-e(k-1)}{T}\right) \tag{10}$$

becomes the output of the PID that is fed into the actuator for a given error. A difference equation for digital PID controller may be derived using transformation formulas such as the Euler backward rule

$$s = \frac{z-1}{zT} \tag{11}$$

Where z-1 is the backward time-shift operator, i.e. x(k-1) =(z-1)x(k). Using this method, the transfer function of a digital PID controller can be written as

$$D(z) = \frac{U(z)}{E(z)} = K\left[1 + \frac{Tz}{\tau_I(z-1)} + \frac{\tau_d(z-1)}{T}\right] \tag{12}$$

2.2 Design of the PID Tuning Algorithm

2.2.1 Determination of Sampling period

The value of the sampling period, T, is directly related to the natural frequency of the system. We obtain the natural frequency , f_n, in Hertz as $f_n = \omega_n / 2\pi$. For proper sampling, the sampling rate f_s must be at least 30 f_n ( ) . For the purpose of this work the sampling rate is set at $40f_n$. Hence,

$$f_s = 40\frac{w_n}{2\pi} \tag{13}$$

Then sampling period T, in seconds is given by

$$T = \frac{1}{f_s} = \frac{2\pi}{40w_n} = \frac{0.157}{w_n} \tag{14}$$

2.3 PID Parameters and Second-order Time Response

Response of a second order system is determined by the value of its damping factor (ξ) and natural frequency (wn). It is hence safe to state that the damping factor and natural frequency determine the characteristic/property of a second order system and consequently the response of the system to an input. The response is a measure of the stability (degree of oscillation) of the system in terms of its damping factor and the speed in terms of the natural frequency(Van der Zalm, 2004).

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When a PID is added to the control system the values of ξ and wn are modified and hence, the property/characteristic of the system is modified as well as its time response specifications in terms of its peak overshoot, rise time , peak time, settling time and delay time The following is the typical structure of a PID controlled second-order system.

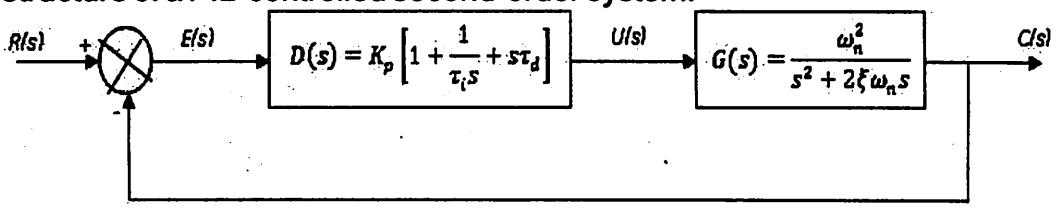

Figure 4: PID controlled second-order system

The transfer function of the PID controller is given by equation (15), that is,

$$D(s) = K_p\left[1 + \frac{1}{\tau_i s} + s\tau_d\right] \tag{15}$$

The open loop transfer function of a second order system in given

$$G(s) = \frac{w_n^2}{s^2 + 2\xi w_n s} \tag{16}$$

The forward transfer function is given by

$$D(s)G(s) = \left(K_p + \frac{K_p}{\tau_i s} + K_p\tau_d s\right)\left(\frac{\omega_n^2}{s^2 + 2\xi\omega_n s}\right) \tag{17}$$

$$D(s)G(s) = \frac{K_p\tau_i\tau_d w_n^2 s^2 + K_p\tau_i w_n^2 s + K_p w_n^2}{\tau_i s^3 + 2\tau_i\xi w_n s^2} \tag{18}$$

Then the closed loop transfer function is

$$\frac{C(s)}{R(s)} = \frac{D(s)G(s)}{1 + D(s)G(s)} \tag{19}$$

$$\frac{C(s)}{R(s)} = \frac{K_p\tau_i\tau_d\omega_n^2 s^2 + K_p\tau_i\omega_n^2 s + K_p\omega_n^2}{\tau_i s^3 + 2\tau_i\xi\omega_n s^2 + K_p\tau_i\tau_d\omega_n^2 s^2 + K_p\tau_i\omega_n^2 s + K_p\omega_n^2} \tag{20}$$

$$\frac{C(s)}{R(s)} = \frac{K_p\tau_i\tau_d w_n^2 s^2 + K_p\tau_i w_n^2 s + K_p w_n^2}{\tau_i s^3 + (2\tau_i\xi w_n + K_p\tau_i\tau_d w_n^2)s^2 + K_p\tau_i w_n^2 s + K_p w_n^2} \tag{21}$$

Dividing numerator and denominator by, gives

$$\frac{C(s)}{R(s)} = \frac{K_p\tau_d\omega_n^2 s + K_p\omega_n^2 + {K_p\omega_n^2}/{\tau_i s}}{s^2 + (2\xi\omega_n + K_p\tau_d\omega_n^2)s + K_p\omega_n^2 + {K_p\omega_n^2}/{K_p\omega_n^2/\tau_i s}} \tag{22}$$

If $\tau_i \gg K_p\omega_n^2$, which is usually the case, ${K_p\omega_n^2}/{\tau_i s} \to 0$. Hence,

$$\frac{C(s)}{R(s)} = \frac{K_p\tau_d w_n^2 s + K_p w_n^2}{s^2 + (2\xi w_n + K_p\tau_d w_n^2)s + K_p w_n^2} \tag{23}$$

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Comparing the above equation to the conventional equation of a second order system, it is observed the value of ω_n and ξ have been modified to w_{npid} and ξ_{pid} respectively given by

$$w_{npid} = w_n\sqrt{K_p} \tag{24}$$

$$\xi_{pid} = \frac{2\xi + K_p\tau_d w_n}{2\sqrt{K_p}} \tag{25}$$

Since w_n and ξ have been modified, the expression for M_p, t_r, t_p, t_s and t_d becomes:

$$t_d = \frac{1+0.7\xi_{pid}}{w_{npid}} \tag{26}$$

$$t_r = \frac{0.8+2.5\xi_{pid}}{w_{npid}} \tag{27}$$

$$t_s = \frac{4}{\xi_{pid} w_{npid}} \tag{28}$$

$$t_p = \frac{\pi}{w_{npid}\sqrt{1-\xi_{pid}^{\,2}}} \tag{29}$$

$$M_p = e^{\frac{\pi\xi_{pid}}{w_{npid}\sqrt{1-\xi_{pid}^{\,2}}}} \tag{30}$$

2.4 Genetic Algorithm Model of the PID Tuner

A genetic algorithm simulator is implemented as a computerized search and optimization procedure that uses principles of natural genetics and natural selection. If a problem-solving mechanism can be represented in a reasonably compact form, then GA techniques can be applied using procedures to maintain a population of knowledge structure that represent candidate solutions, and then let that population evolve over time through competition (survival of the fittest and controlled variation) (Pereira, 2005). The GA will generally include the three fundamental genetic operations of selection, crossover and mutation. These operations are used to modify the chosen solutions and select the most appropriate offspring to pass on to succeeding generations. The flow of the implemented GA is presented in Figure 5.

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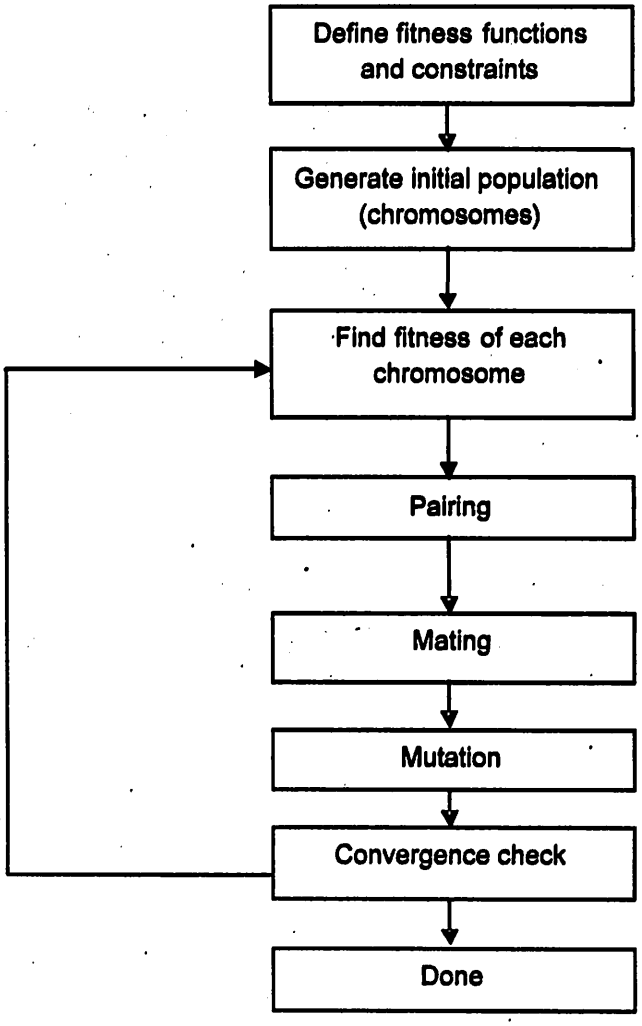

Figure 5: A block diagram of the Genetic Algorithm process

2.5 Tuner Design

The aim is to obtain the corresponding Proportional (K_p), Integral (τ_I) Derivative (τ_d) gains of the controller to desired values of t_p, t_r, t_p, t_s And t_d To achieve this, the genetic algorithm process shown in figure 8 is followed. In the beginning, the fitness function is defined, and then an initial chromosome population is randomly generated. The chromosomes are candidate solutions to the problem, then, the fitness values of all chromosomes are evaluated. Based on the fitness of each individual, a group of the best chromosomes is selected through the selection process. The genetic operators, crossover and mutation, are applied to this "surviving" population in order to improve the next generation solution. Crossover is a recombination operator that combines subparts of two parent chromosomes to produce offspring. This operator extracts common features from different chromosomes in order to achieve even better solutions. Mutation is an operator that introduces variations into the chromosome and occurs occasionally with a small probability. It randomly alters the value of a bit, in case of binary coding.

In real coding it changes the entire value of a chromosome.Through the mutation operator the search space is explored by looking for better points. The process continues until the population converges to the global maximum or another stop criterion is reached (Haupt and Haupt). A chromosome is modelled as []. The values of, and are calculated for each chromosome and compared to their desired values. The closer the calculated values are to the desired values, the higher the fitness of the chromosome. Therefore, the fitness function is expressed as:

$$F(x) = \frac{1}{\left[(Exp\_t_r - Cal\_t_r) + (Exp\_t_s - Cal\_t_s) + (Exp\_t_p - Cal\_t_p) + (Exp\_t_d - Cal\_t_d) + (Exp\_M_p - Cal\_M_p)\right]}$$

where $Exp\_t_r = Expected\ value\ of\ t_r$; $Exp\_t_s = Expected\ value\ of\ t_s$;
$Exp\_t_p = Expected\ value\ of\ t_p$; $Exp\_t_d = Expected\ value\ of\ t_d$;
$Exp\_M_p = Expected\ value\ of\ M_p$; $Cal\_t_r = Calculated\ value\ of\ t_r$;
$Cal\_t_s = Calculated\ value\ of\ t_s$; $Cal\_t_p = Calculated\ value\ of\ t_p$;
$Cal\_t_d = Calculated\ value\ of\ t_d$; $Cal\_M_p = Calculated\ value\ of\ M_p$

3.0 MODEL SIMULATION

Based on the operation of genetic algorithm to solve the tuning of digital P-I-D controller for Second-Order Systems, the following step-by-step approach has been adopted:

Step 1: Start with a randomly generated population of n chromosomes (generated population must be candidate solution to the problem)

Step 2: Calculate the values of M_p, t_r, t_p, t_s and t_d for each chromosome in the population.

Step 3: Calculate the fitness $F(x)$ of each chromosome in the population

Step 4: Rearrange the chromosomes in descendi ng order of fitness so that the individuals with high fitness are on top (Ranking)

Step 5: Use the selection rate (X_{rate}) to dete rmine the number of chromosomes to keep for the mating pool; use X_{rate}=50% (Selection)

Step 6: Using the "pairing from top to bottom" method, pair chromosomes two at a time until every chromosome in the mating pool has been paired for mating. (Pairing)

Step 7: Randomly select a kinetochore or crossover point between the first (K_p) and the last value (τ_d) of the parents' chromosomes (different kinetochore can be used for different pairs of parent)

Step 8: Mate parents to produce offspring equal to the number of parents. (Mating)

Step 9: Calculate the values of M_p, t_r, t_p, t_s and t_d for each chromosome in the population.

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Step 10: Calculate the fitness of each individual in the new population
Step 11: Rearrange the new population in descending order of fitness so that the individuals with high fitness are on top (Ranking)
Step 12: Mutate the new population leaving only the elite
Step 13: Rearrange the population in descending order of fitness so that the individuals with high fitness are on top (Ranking)
Step 14: Repeat step 5 to step 13 k-times (where k = number of generations)
Step 15: Output fittest chromosome.

The genetic algorithm process was coded using *Java programming language* (Sanchez and Canton, 2002) . This will enable fast and less cumbersome task when evaluating large number of generations.
Let the system have the following system operating parameters:

- Natural Frequency = 173.8192rad/s
- Damping Ratio = 0.31874

For these simulations, the following data are important:
Population= 8
Chromosome= $[K_p \tau_i \tau_d]$
Lower bound= [0.1, 0.1, 0.005]
Upper bound= [50, 0.5, 0.0001]
Generations= (variable)
Mutation rate= 0.3

4.0 SIMULATION RESULTS

The simulation consists of two parts. The first part is verifying the efficiency and accuracy of the solution using the number of generations. The results are as shown in Table 1.

Inference: It shows that the calculated performance parameters get closer to the expected performance parameters as the number of generations increases. The process follows the GA theory that the probability of searching every point in the solution space increases as the number of generation increases.

The second part of the simulation consists of instances of different values of the performance parameters, and their corresponding results are given in Table 2. . $(K_p$ (τ_i) (τ_d) .
Inference: These results show a very small difference between the calculated performance parameters and expected values. In other words, the GA attempts to obtain values of Proportional, Integral and Derivative gains of the controller which yields performance parameters with minimal deviation for the expected values. If the number of generations

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Table 1: Results verifying the efficiency and accuracy of the GA

Controller Type	Expected Performance Parameters					Controller Parameters(Chromosome)			Calculated Performance Parameters					Number of generations
	t_r (s)	t_d (s)	t_s (s)	t_p (s)	M_p	K_p	T_I (s)	T_D (s)	t_r (s)	t_d (s)	t_s (s)	t_p (s)	M	
None	0.00919	0.00704	0.07219	0190	0.34768			X	0.00919	0.00704	0.07219	0.01907	0.34768	
	0.0163	0.0109	0.067	0.029	0.98605	1.5423	0.3074	0.00028	0.007	0.0055	0.0646	0.0152	0.9956	10
	0.0163	0.0109	0.067	0.029	0.98605	0.5443	0.2483	0.000228	0.0149	0.0102	0.0698	0.0274	0.9878	100
	0.0163	0.0109.	0.067	0.029	0.98605	0.4955	0.4761	0.00034	0.0162	0.0109	0.0691	0.0291	0.986	.1000
Proportional	0.0163	0.0109	0.067	0.029	0.98605	0.5135	0.1574	0.0007	0.0162	0.0108	0.0657	0.0289	0.9859	10000
Plus Integral Plus	0.0163	0.0109	0.067	0.029	0.98605	0.4820	0.4659	0.00013	0.0162	0.01099	0.07102	0.0294	0.986	100000
Derivative	0.0163	0.0109	0.067	0.029	0.98605	0.4933	0.3117	0.00031	0.0162	0.0109	0.0693	0.0291	0.986	1000000

Table 2: Results of instances of different values of the performane parameter

Controller Type	Expected Performance Parameters					Controller Parameters(Chromosome)			Calculated Performance Parameters				
	t_r (s)	t_d (s)	t_s (s)	t_p (s)	M_p	K_p	$_I$ (s)	T_D (s)	t_r (s)	$_d$ (s)	(s)	$_p$ (s)	M_p
None	0.00913	0.00699	0.0718	0.019	0.34768	X		X	0.00913	0.00699	0.0718	0.019.	0.34768
	0.0163	0.0109	0.067	0.0291	0.986	0.4958	0.4174	0.0004	0.0162	0.0109	0.068	0.0292	0.986
	0.00735	0.00543	0.0489	0.0146	0.9948	1.684	0.1427	0.00048	0.006	0.0054	0.059	0.0146	0.9956
	0.0032	0.0023	0.0194	0.0062	0.9976	12.0417	0.2529	0.0008	0.0027	0.0021	0.0194	0.0055	0.9981
Proportional Plus Integral	0.0039	0.0031	0.0345	0.0085	0.9974	5.8594	0.3446	0.00068	0.0035	0.0028	0.034	0.0077	0.9978
Plus Derivative	0.0051	0.0043	0.0599	0.0117	0.997	2.4701	0.4967	0.00034	0.0052	0.0043	0.0586	0.0118	0.9970

5.0 CONCLUSION

This paper demonstrated how genetic algorithm can be used to obtain the corresponding Proportional constant (K_p) Integral time(τ_I) and Derivative time(τ_d) to give desired values of the performance parameters (i.e., M_p , t_r, t_p, t_s and t_d).).

A PID (three-term) controller also has the capability of restricting a plant to behave in certain desired manner when properly tuned and the discretizaion of PID and/or control systems afford engineers the opportunity to implement the PID controller using a digital computer.

Although a number of works have been carried out on Genetic Algorithm-

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