Comparison of Errors Caused by Flux Limiters on the Numerical Solution of Advection-Diffusion Problem

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ABSTRACT: Flux limiters are widely used in numerical simulations to prevent spurious oscillation in the flow with strong property gradients. However, applying flux limiter on flow without strong property gradient such as advection-diffusion flow can cause errors. This article discusses the errors caused by several flux limiters in advection-diffusion flow solution. A method for applying one-dimensional limiters to two-dimensional unstructured mesh was also suggested. The error was measured by comparing the finite volume solution of a test case with a reference solution. The study shows that the calculation error of second-order finite volume with flux limiter was higher than that of second-order finite volume without limiter. However, the error of third-order finite volume with flux limiter is less than that of second-order without flux limiter. Among the flux limiters tested in this study, Venkatakrishnan's flux limiter produces the highest error, followed by Van leer's limiter, EULER and SMART limiter.

KEYWORDS: Flux limiter, Monotone, Unstructured mesh, Numerical solution, Errors, Accuracy

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I. INTRODUCTION

The accuracy of the finite-volume solution of the transport equation is strongly influenced by the interpolation of flow variables from the cell centre to the face centre of finite volume cells. First-order interpolation does not produce sufficiently accurate results in most practical and industrial cases due to the high diffusive error. Second-order interpolation is able to produce a more accurate solution but often results in spurious oscillations in the regions which have strong property gradients.

Numerous affordances have been developed to reduce the spurious oscillation during the last decade to solve various problems arising in mathematical physics. The quest to compute sharper shock waves without spurious oscillation was based on flux limiting, where the interpolated fluxes at control volume faces are obtained as the sum of the first order accurate flux plus a fraction of correction needed to make the flux second-order accurate. This partial correction is chosen to be as large as possible whilst not producing solution oscillations. The criterion established by Harten (1983) for the avoidance of oscillations was that the total variation of convected properties should diminish at each time step. Such methods are therefore known as Total Variation Diminishing (TVD) methods. The general theory of TVD methods was developed from the early 1970s by many workers, including Van Leer (1974), Boris and Book (1973), Chakravarthy and Osher (1983), Sweby (1984) and Leonard (1983). By the late 1980s, the method was firmly established for structured mesh gas dynamics and aerodynamics codes and was being advocated for more general Computational Fluid Dynamics (CFD) application by authors such as Leonard and Mokhtari (1990) and Gaskell and Lau (1988). Along with its popularity, the classic TVD limiters have been improved by many researchers. Recently, Tang and Li (2020) improved the van Albada and van Leer limiters to reduce dissipation in numerical solutions. Govind and Nair (2022) developed a high-order limiter for the Euler equation numerical solution. Cai et al. (2021) proposed a new slope limiter for the hyperbolic conservation law equation. The limiter can produce a high resolution and non -oscillatory numerical solution.

Almost the entire the early TVD scheme was for one-dimensional cases. The famous 2D and 3D TVD scheme for hyperbolic conservation law on the structured grid was then developed by Spekreijse (1987). Base of the work of Spekreijse, Barth and Jespersen (1989) developed TVD for 2D and 3D unstructured grid. Later the work of Barth and Jespersen was improved by Venkatakrishnan (1993) to reduce the unnecessary reduction in accuracy when Barth and Jespersen's limiter is applied to cells with more than three faces and to improve solution convergence. Jameson (1994) claimed that TVD is a one-dimensional concept where it cannot be applied to multidimensional cases. He proposed a monotone scheme for multi-dimensional cases, so-called Local Extremum Diminishing (LED), but this scheme is computationally expensive due to the use of vertex-based finite volume. Bruner (1995), Darwish and Moukalled (2003) and Tasri (2005, 2021) separately used TVD scheme for unstructured grid based on the one-dimensional TVD scheme. Lochap and Kumar (2021) improved the two-dimensional flux limiter using a fuzzy modifier for use in hyperbolic conservation flow. Recently, Valle et al. (2022) developed a method for applying one-dimensional limiters to a two-

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dimensional case that allows easy implementation in an algebraic framework.

To find the best among the limiters, Juntasaro and Marquist (2003) compared these limiters to determine the most accurate by using advection flow as a test case. Darwish and Mukalled (2003) and Tasri (2021) compared the application of several one-dimensional limiters to an unstructured mesh finite volume scheme using a double step advection flow as test case. Similar to Darwish and Mukalled (2003) and Tasri (2021), Wang et al. (2013) studied the performance of some limiters using advection flow test cases. Kivva (2020), Wu et al. (2017) and Zang et al. (2015) compared some newly developed flux limiters for a convective-diffusive equation using an advection flow test case. Govin and Nair (2022) compared several flux limiters with a new high-order slope limiter using supersonic flow through a wedge as a case study. Li (2020) investigated the compressive properties of the Min-mod-type limiter and compared the performance of the limiter with several TVD limiters. It found that compares to the TVD limiter; the Min-mod limiter tends to be more attractive in modelling shockwave-containing flows due to its stable computational process.

Most flux limiters were tested on flows with strong properties gradient. The application of limiters in the solution of flow that does not have a strong property gradient has not received much attention. Although, using a flux limiter in the flow without strong property gradient can cause errors as the limiter cuts the high-order term of the equation used to interpolate the flow variable from the cell centre to the face centre. The use of limiters for this type of flow is widespread in practical applications because it is not applicable to create a numerical solver that can be used for flows with strong properties gradient or without strong properties gradient only.

In this work, the effect of flux limiters on the error of the finite volume solution of flow without strong properties gradient was studied. A method for applying one-dimensional limiters to two-dimensional unstructured mesh was also suggested. A conventional advection-diffusive flow in a backwards-facing step was used as a test case.

II. TVD LIMITER

The main idea of TVD is that numerical solution will be oscillation free if Total Variation does not increase during iteration (Harten, 1983). Roe (1981) applied the TVD scheme in finite volume methods by write the face value of cell centre variable, ϕ_f, as sum of a diffusive first order upwind term and a limiter multiplied, a higher order, anti-diffusive term. The limiter is a nonlinear function of variable *r*, which measure local smoothness. The local smoothness *r* was introduced by Van Leer (1974) for 1D cases as in Eqn. (1),

$$r = \frac{\phi_C - \phi_U}{\phi_D - \phi_C} \tag{1}$$

leading to limited scheme of cell centre to face centre interpolation of flow variable.

$$\phi_f = \phi_C + \frac{1}{2}\psi(r)(\phi_D - \phi_C) \tag{2}$$

$\psi(r)$ is a limiter function. ϕ_D is downwind cell variable of face *f*. ϕ_C and ϕ_U are upwind and far upwind cell variable as shown in Figure 1.

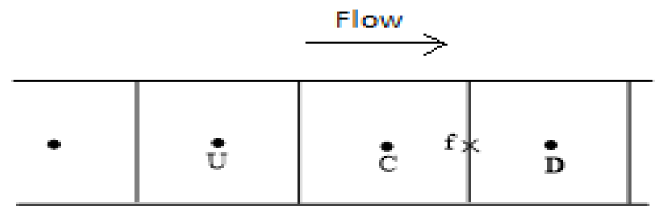

Figure 1: Upwind and downwind cell of one-dimensional mesh.

There are many equations for the limiter as found in the literature. Two of the limiters studied in this work are presented in Eqns. (3) and (4).

Superbee limiter:

$$\psi(r) = \begin{cases} 0 & \text{if} \quad r < 0 \\ 2r & \text{if} \quad 0 \le r \le 0.5 \\ 1 & \text{if} \quad 0.5 \le r \le 1 \\ r & \text{if} \quad 1 \le r \le 2 \\ 2 & \text{if} \quad r > 2 \end{cases} \tag{3}$$

Van Leer limiter:

$$\psi(r) = \begin{cases} 0 & \text{if } r < 0 \\ \dfrac{2r}{1+r} & \text{if } r \ge 0 \end{cases} \tag{4}$$

An alternative but related flux limiter approach for obtaining monotone solutions was developed independently by Leonard (1983, 1990). Leonard defined a normalised variable to replace local smoothness *r*. The normalised variable ϕ associated with a cell centre *C* defined as in Eqn. (5).

$$\hat{\phi}_C = \frac{\phi_C - \phi_U}{\phi_D - \phi_U} \tag{5}$$

Two example of specimen TVD scheme for flux of flow variable based of normalised variable of Leonard are as in Eqns. (6) and (7).

Gaskell's SMART limiter, given by:

$$\phi_f = \begin{cases} 3\hat{\phi}_C & \text{if } 0 < \hat{\phi}_C < \dfrac{1}{6} \\ \dfrac{3}{8} + \dfrac{3}{8}\hat{\phi}_C & \text{if } \dfrac{1}{6} < \hat{\phi}_C < \dfrac{5}{6} \\ 1 & \text{if } \dfrac{5}{6} < \hat{\phi}_C < 1 \\ \hat{\phi}_C & \text{elsewhere} \end{cases} \tag{6}$$

Leonard's EULER limiter, given by:

$$\phi_f = \begin{cases} \dfrac{\sqrt{\hat{\phi}_C(1-\hat{\phi}_C)^3} - \hat{\phi}_C{}^2}{1-2\hat{\phi}_C} & \text{if } 0 < \hat{\phi}_C < 1 \\ 0.75 & \text{if } \hat{\phi}_C = 0.5 \\ \hat{\phi}_C & \text{elsewhere} \end{cases} \tag{7}$$

The original TVD scheme is only derived for a one-dimensional or structured mesh. The application of the one-dimensional TVD limiter on an unstructured mesh is not straightforward. In the arbitrary unstructured meshes, it is unclear how to determine the far upwind cell *U* since the mesh does not have any clear directionality, as shown in Figure 2. For example, any points at the upwind of cell *C* can be considered as far upwind cell centre of face *f*. Another problem in applying one-dimensional limiter to unstructured mesh is cells at the upstream and downstream of face *f* do not conjunctional and orthogonal to face *f*. This condition caused a diffusive error (Wang et al., 2013). To overcome this problem, auxiliary points *C'* and *D'* were created to replace upwind and downwind cell centres, *C* and *D*, plus a third point *U'* upstream of *C'* such that $\mathrm{r}_{C'U'} = -\mathrm{r}_{C'D'}$ and $\overline{C'f} = 0.5\,\overline{CD}$ as shown in Figure 2. $\mathbf{r}_{C'U'}$ was a distance vector from *C'* to *U'* and $\overline{C'f}$ was distance from *C'* to face *f*. Since numerically calculated value $\nabla\phi$ at the point *C'* can be found using Least Square methods (Tasri, 2022), ϕ at fictitious point *U'* may be found with second order accuracy from:

$$\phi_{U'} = \phi_{D'} - 2\nabla\phi_{C'} \cdot \mathrm{r}_{C'D'} \tag{8}$$

where

$$\phi_{D'} = \phi_D - \nabla\phi_D \cdot \mathrm{r}_{DD'} \tag{9}$$

$$\phi_{C'} = \phi_C - \nabla\phi_C \cdot \mathrm{r}_{CC'} \tag{10}$$

Once ϕ at the fictitious cells are known then the standard one dimensional TVD methods may be used to determine smoothness monitor of *r* in Eqn. (1) and hence the TVD limited face variable for face *f* as in Eqn. (2).

$$r = \frac{\phi_{C'} - \phi_{U'}}{\phi_{D'} - \phi_{C'}} \tag{11}$$

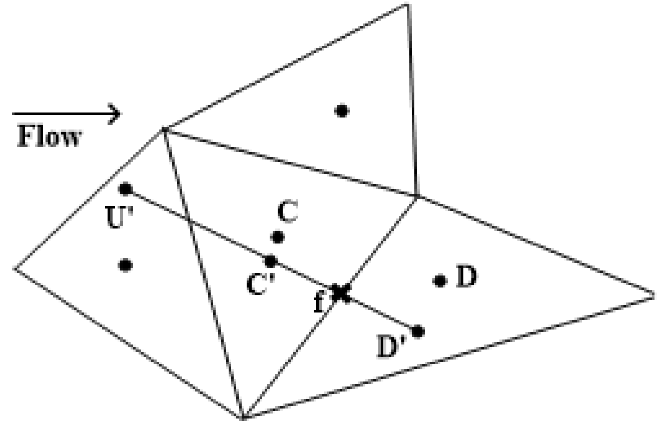

Figure 2: Spatially corrected cell centres and fictitious cell.

The method is considerably less restrictive than those of Barth and Jespersen's limiter (Barth and Jespersen, 1989). Firstly, the limiting action is applied face by face, rather than having common limiter for all downstream faces of the cell as in Barth and Jespersen's limiter. Secondly, the user has complete freedom of choice from the wide range of well-tested structured mesh limiter schemes available.

Venkatakrishnan (1983) improved Barth and Jespersen's limiter by using the reconstruction method to calculate the face value of variables so that the value of convected scalar ϕ at face *f* of cell was obtained from the cell centre value ϕ_C and the cell centre gradient $\nabla\phi_C$ using

$$\phi_f = \phi_C + \psi\nabla\phi_{\mathbf{C}} \cdot \mathbf{r}_{\mathbf{Cf}} \tag{12}$$

where r_{Cf} is a vector from cell centre to centre of face. ψ is a limiter to limit gradient $\nabla\phi_{\mathrm{C}}$ to satisfy TVD condition. The TVD condition require the linear reconstruction at any point within or on boundary cell *C*, $\phi(x, y)$ should be bounded by ϕ at neighbour cell.

$$\phi_{Min} \le \phi_{(x,y)} \le \phi_{Max} \tag{13}$$

where

$$\phi_{Min} = Min.(\phi_C, \phi_{neighbour}) \tag{14}$$

$$\phi_{Max} = Max(\phi_C, \phi_{neighbour}) \tag{15}$$

$\phi_{neighbour}$ denotes the cell centre value at all immediate face neighbour cells to cell *C*.

The limiter ψ is calculated as:

$$\psi = Min(\psi_f) \tag{16}$$

Where ψ_f was calculated from:

$$\psi_f = \frac{y^2 + 2y + \varepsilon^2/(\phi_{f\,i} - \phi_P)}{y^2 + y + 2 + \varepsilon^2/(\phi_{f\,i} - \phi_P)} \tag{17}$$

$$y = \begin{cases} \left(\dfrac{\phi_{Max} - \phi_C}{\phi_{fi} - \phi_C}\right) & \Leftarrow \phi_{fi} - \phi_C > 0 \\ \left(1, \dfrac{\phi_{Min} - \phi_C}{\phi_{fi} - \phi_C}\right) & \Leftarrow \phi_{fi} - \phi_C < 0 \\ 1 & \Leftarrow \phi_{fi} - \phi_C = 0 \end{cases} \tag{18}$$

The unlimited estimation for ϕ_{fi} was given by

$$\phi_{fi} = \phi_C + \nabla\phi \cdot \vec{r}_{Cf} \tag{19}$$

The small quantity ε in Eqn. (17) was determined, somewhat arbitrary as:

$$\varepsilon = (\kappa \Delta x)^3 \tag{20}$$

where κ is an empirical constant, typically 0.3, and Δx is mean grid spacing.

III. LIMITER COMPARISON

This study used a laminar advection-diffusion flow in a two-dimensional backwards-facing step as a test case. The backwards-facing step has a step high equal to duct inlet H. An inlet section of length 4H was used to allow a laminar velocity profile developed upstream of the step. A constant pressure outlet boundary was located at 10H downstream of the step, well downstream of the expected re-attachment length.

The Reynolds number of the flow, based on H and average inlet velocity, was 100. This data was originally used by Peric et al. (1988) to compare finite volume solutions obtained using collocated and staggered, structured meshes.

Based on Richardson's interpolation idea (Burg and Erwin, 2009), the reference solution for this case was generated using numerical software Fluent with a fine mesh of 60000 square cells. Second-order upwind differencing was used for convected velocity components. Fluent uses an approximate linear interpolation for the convection velocity components, but for the square cells used here both this and the pressure interpolation should be truly second-order accurate. The solution was run to convergence limit of 10^{-6} for normalised continuity residual, 3 orders of magnitude lower than Fluent's default convergence criterion.

The test solutions for Van Leer, EULER and SMART limiter, as well as a non-limited scheme, were determined using a computer code developed during this study. While test solution for Venkatakrishnan's limiter test case was calculated using ANSYS fluent, which used the limiter used as a default limiter.

The test solutions were calculated using an unstructured mesh of 5314 triangular cells, generated using equal face lengths on all boundaries, with ten faces along the duct inlet as shown in Figure 3. SIMPLE algorithm of Patankar and Spalding (Patankar, 1972) and momentum interpolation of Rhie and Chow (Rhie, 1983) based scheme were used to pressure correction and preserve pressure and velocity coupling. Face value of field variable was calculated using a second-order upwind biased scheme. The gradient at the cell centre was calculated using Gauss's theorem. The no-slip boundary condition is specified on the wall surface. A uniform velocity profile at the inlet was used. Uniform pressure distribution is specified at the outlet boundary. In each case, the reference fine grid solution fields were subtracted from the test solution fields, and differences were plotted to aid comparison.

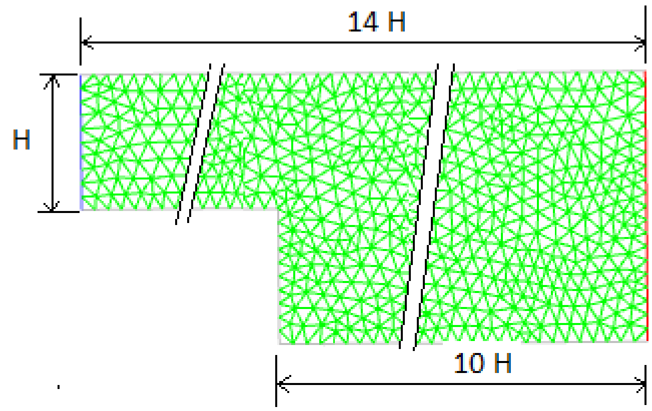

Figure 3: Backward facing step domain and mesh.

The contours of dimensionless *x*-velocity errors for several flux limiters, are shown in Figure 4. Figure 4 (a) and (b) shows the error of the limited second-order scheme of Venkatakrishnan (Venkatakrishnan, 1993) and Van Leer (Van Leer, 1974) flux limited schemes, respectively. Figure 4 (c) and (d) show the error of Leonard's EULER scheme (Leonard, 1983) and the third-order SMART scheme of Gaskell and Lau (Gaskel, 1988). The limiters are intended to increase accuracy by preventing spurious oscillations in locations with strong properties gradient. However, if the limiters are used in region without a strong properties gradient, such as in the advection-diffusion flow in backward facing step, it causes a decrease in accuracy as shown in Figure 4 and Table 1. The figure and table show second-order finite volume with Venkatakrishnan's, Van Leer's and EULER limiter has a higher error than the unlimited second-order finite volume scheme. This condition was possible because of the action of the limiters to limit the anti-diffusion term of the equation for interpolation of flow variable from cell centre to face centre to satisfied TVD condition. However, the third-order finite volume with the SMART limiter looks better than the second-order unlimited scheme.

Figures 4 and 5 show that Venkatakrishnan's limiter failed in the shear layer and uniform velocity region. It was, suspected to be caused by the action of limiting and attenuating the gradient in all directions equally, even in the area where no extremes are formed, as mentioned by Michalak (2008) and Tasri (2005). The choice of limiter function was restricted, and the use of a single limiter value for all downstream faces, as in Venkatakrishnan's limiter, may be overly restrictive. It is important to note that, although superficially, the

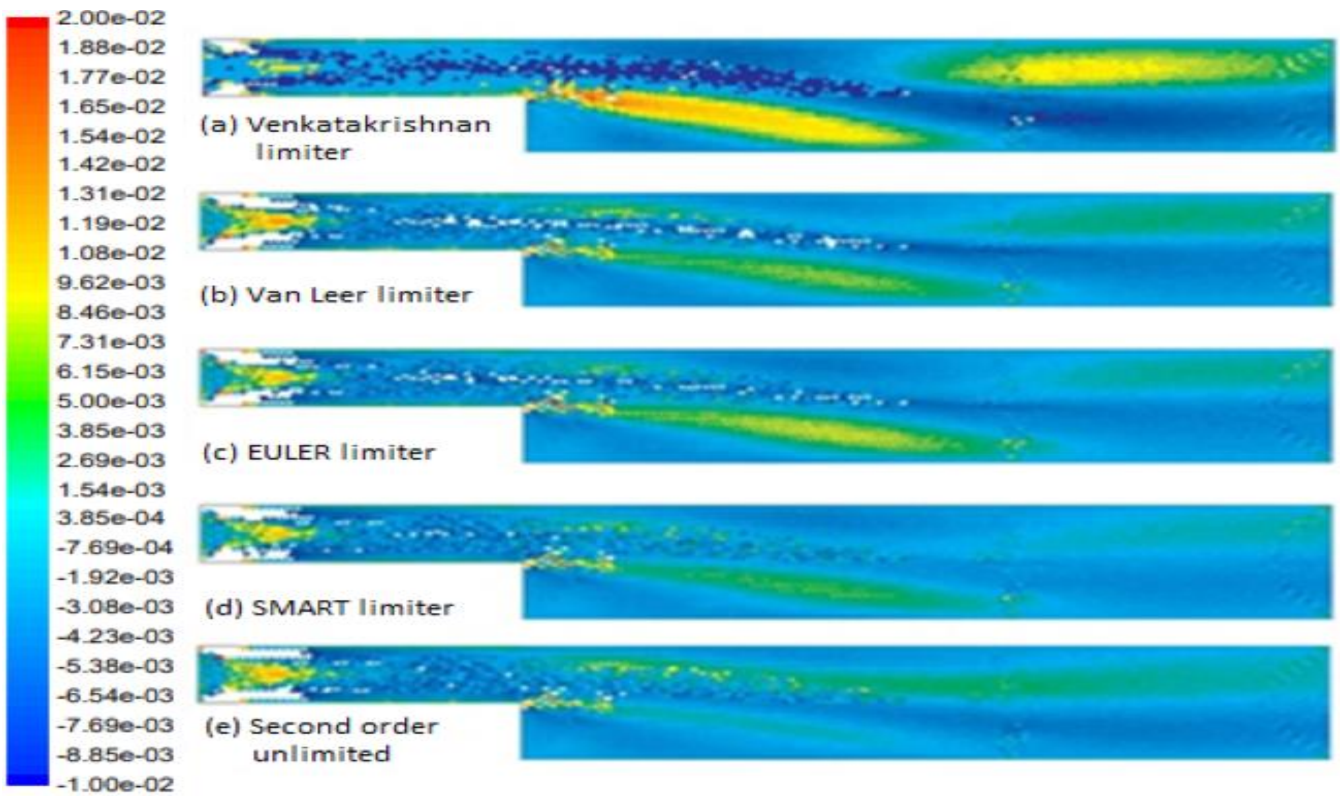

Figure 4. Effect of flux limiters on x-velocity error. The error was calculated as the difference between the reference solution and the test solution.

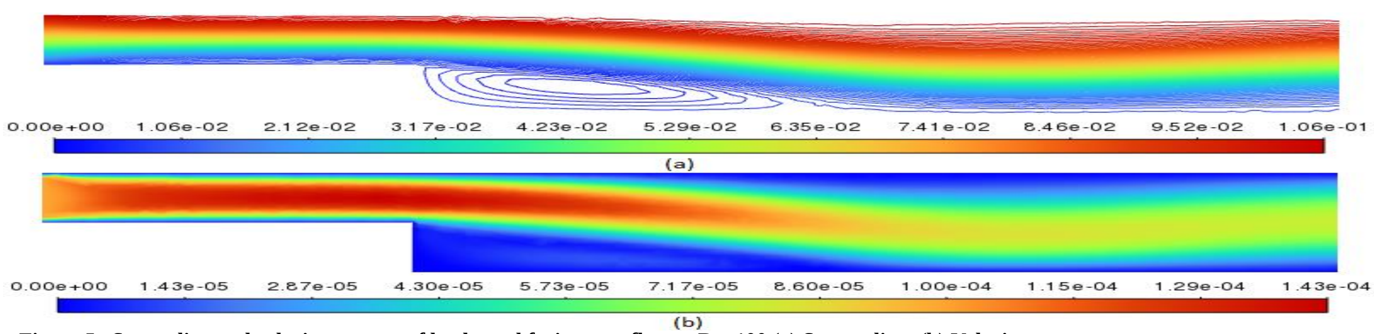

Figure 5. Streamline and velocity contour of backward facing step flow at Re=100 (a) Stream line; (b) Velocity contour.

Table 1. Effect of flux limiter on L1 norm error of x-velocity.

Limiter and Interpolation methods	*L1* error
Venkatakrishnan limiter	0.004822
Van Leer limiter	0.002838
Leonard's Euler limiter	0.002728
Gaskell's Smart limiter	0.002073
Without limiter	0.002282

Venkatakrishnan method was based on the higher accuracy 2D Taylor series expansion, the interpolative accuracy is lower than for a linear interpolation, even for smooth variation of field variable. Not only do the test function of Equation (16), used to determine the limiter value, depend upon the same approximations as a linear interpolation between cell centres, but the comparator values ϕ_{Max} and ϕ_{Min} is not apply directly to the current face. Finally, the need to add a switching function to remove limiter action in regions of near-constant, as in Venkatakrishnan's modification, somewhat weakens the initial simplicity and elegance of the method.

It is well known that achieving smooth transition between discontinuous jumps with first-order representation and a sharp but continuous gradient requires second-order consistency, apart from its ability to avoid convergence to stall to steady-state. So that, the differentiable limiters, such as Van Leer limiter perform much better in the area of non-uniform velocity. The methods are also considerably less restrictive than those of Venkatakrishnan's due to the limiting action is applied face by face, rather than having a similar limiter for all the downstream faces of a cell.

Leonard's EULER scheme uses a continuous limiter function to improve convergence behaviour at the expense of slightly more deviation from the base third-order scheme. EULER scheme, though nominally third-order accurate, performs little better than Van Leer limiter in uniform velocity region.

SMART scheme of Gaskell and Lau (Gaskell, 1988) uses a discontinuous limiter function to more closely approximate Leonard's third-order Quadratic Interpolation for Convective Kinetics (QUICK) interpolation (Leonard, 1983) where possible. Because of its third-order approximation, the SMART limiter performs better among the limiter tested here. It cannot avoid the error in non-uniform velocity regions completely, as shown in Figure 4d. This must represent the closest possible approach of a flux-limited scheme to its unlimited counterpart, in terms purely of interpolative accuracy, and may serve as one yardstick for evaluating alternative schemes.

IV. CONCLUSION

The effects of flux limiters on accuracy of the finite volume solution of advection-diffusion flow were studied in this work. The main findings can be summarized as follows:

i) Flux limiter is useful to prevent spurious oscillation in the region with strong properties gradient. Still, it causes a decrease in accuracy in the regions that do not have a strong property gradient. In the advection-diffusion flow test case, the error of second-order finite volume with Venkatakrishan, EULER and Van Leer flux limiter was higher than the error of second-order scheme with no limiter. However, the error in third-order finite volume with SMART flux limiter is less than second-order finite volume without flux limiter.
ii) Among the flux limiters tested in this study, the Venkatakrishnan limiter produces the highest error, followed by Van Leer, EULER and SMART limiter.

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AUTHOR CONTRIBUTIONS

A. Tasri: Developed a method for applying one-dimensional limiters to two-dimensional unstructured mesh; Conceived and designed numerical experiments; performed the experiments; Analysed and interpreted the data; Wrote the paper.

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